Developing Fragility Functions for Building Components for ATC-58

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1 INTRODUCTION

1.1 OBJECTIVE AND ORGANIZATION OF THIS DOCUMENT

This text addresses the analysis of damage data to create and test fragility functions for building components. Damage data may be empirical, analytical, or from expert opinion. The data comprise knowledge of the engineering demand parameter (*EDP*) to which components are subjected, as well as the associated damage. Damage is defined in terms of required repairs and other consequences. Here, a component fragility function means the probability that a component of a given type will reach or exceed a particular damage state, denoted by *dm*, as a function of *EDP*. The word "failure" is used throughout this text to refer to the condition of reaching or exceeding a specified damage state. The text primarily addresses fragility functions idealized by the lognormal cumulative distribution function.

Section 1 introduces the problem. Mathematical methods for developing fragility functions are presented in Section 2. Section 3 deals with data outliers and assigning quality levels to fragility functions. Examples are presented in Section 4. Section 5 discusses damage correlation between different components in a building. Sections 6 and 7 contain acknowledgements and references, respectively.

1.2 DOCUMENTATION REQUIREMENTS

Following are recommended requirements to document source data, analysis, and resulting fragility functions. Document all source data, including the following.

1. *Bibliographic reference* of any primary source.
2. *Description of specimens*. Describe what was tested or observed, its materials and quantitative material properties (where available), configuration, building code or code era if known and applicable, number of specimens observed, and location of tests or observations.
3. *Excitation*. Loading protocol or characteristics of earthquake motion.
4. *Damage evidence.* Kinds of physical damage or force-deformation observed.
5. *Summary of observations.* Tabular or graphical data listing specimens, excitation to which each was subjected, and damage evidence for each.

Document the analysis of the fragility function, including a least the following:

1. *Analysis method.* Identify the method used to derive the fragility function, from Table 1-1.
2. *Excitation to EDP.* Method of inferring *EDP* from loading protocol or observed excitation. Indicate whether *EDP* is the value at which damage occurred (method A data) or maximum each specimen experienced (methods B, C, and U). Table 1-3 lists some *EDP*s in use, the first five of which are most likely to be used in ATC-58.
3. *Damage evidence to damage state*. Method of inferring *dm* from physical damage or force-deformation observations.
4. *Summary of inferred EDP and dm data.* Tabular or graphical data listing specimens, *EDP*, *dm*.
5. *Show your work*. Provide sample calculations. Do not skip steps.

Document the results of the analysis, providing all the information listed in Table 1-2. The present work focuses on how to estimate x_m and β using various kinds of data.

Table 1-1. Analysis methods and required data

Method name	Data required
A. Actual failure *EDP*	All specimens failed at observed values of *EDP*.
B. Bounding *EDP* with damage	Some specimens failed. Maximum *EDP* to which each specimen was subjected is known.
C. Capable *EDP*	No specimens failed. Maximum *EDP* to which each specimen was subjected is known. E.g., seismic qualification tests
D. Derived fragility	Fragility functions produced analytically by reliability methods
E. Expert opinion	No data available; expert judgment is required
U. Updating	Enhancing an existing fragility function with new failure data

Table 1-2. Features of a well defined fragility function

Feature	Comment
Taxonomic group	Define the component using the ATC-58 taxonomy
Units	Select the units in which the component is counted (square feet, number, etc.).
DM	Define the damage state quantitatively in terms of repairs required to restore the component. If repair efforts are uncertain, estimate means and standard deviations. Identify failure consequences: repair cost (Y/N), threaten life-safety (Y/N), loss of use (Y/N).
EDP	Identify the *EDP*(s) most closely related to failure probability. Competing possible *EDP*s are addressed in Section 3.1.
x_m and β	Parameters of the fragility function

Table 1-3. Some EDPs in use

EDP	**Definition**	**Ref**
PTD	Peak transient drift ratio (specified for a story, column line, and direction). $PTD = \lvert x_i - x_{i-1}\rvert / h_i$, where x_i = displacement of floor i, x_{i-1} = displacement of floor i-1, h_i = height of story i from top of finished floor to top of finished floor, and the ratio is taken as the maximum over time during seismic loading.	
PDA	Peak diaphragm acceleration, max horizontal direction (specified for story)	
RD	Residual drift ratio (specified for story, column line, and direction). $PTD = \lvert x_i - x_{i-1}\rvert / h_i$, where x_i = displacement of floor i, x_{i-1} = displacement of floor i-1, h_i = height of story i from top of finished floor to top of finished floor, and the ratio is taken after the end of seismic loading.	
MHR	Maximum hinge rotation, rad	
G	Maximum shear strain	
CDR	Curvature ductility ratio CDR = $(f_m - f_y)/(f_u - f_y)$, where f_r = recoverable curvature of flexural member (i.e., curvature when yield moment occurs), f_u = curvature of flexural member when ultimate moment occurs, f_m = maximum curvature of flexural member attained during seismic loading	
PADI	Modified Park-Ang Damage Index; specified for member and direction PADI = $(f_m - f_y)/(f_u - f_y) + b(A_t/(f_u M_y))$, where f_r = recoverable curvature of flexural member (i.e., curvature when yield moment occurs), f_u = curvature of flexural member when ultimate moment occurs, f_m = maximum curvature of flexural member attained during seismic loading, b = strength deterioration parameter, A_t = total area contained in M-f loops, and M_y = yield moment of flexural member.	[1]
DC	Peak transient drift ratio at beam-column connection, in percent $DC = 100\lvert x_a - x_b\rvert/(0.5(h_a + h_b))$, where x_a = displacement of the mid-height of the column above the connection, x_b = displacement of the mid-height of the column below the connection, h_a = height of the column above the connection, and h_b = height of the column below the connection	

1.3 DEFINITION OF FRAGILITY FUNCTIONS AND PROBABILISTIC DAMAGE STATE

Let $F_{dm}(edp)$ denote the fragility function for damage state *dm*, defined as the probability that the component damage state reaches of exceeds *dm*, given a particular *EDP* value, i.e.,

$$F_{dm}\left(edp\right) \equiv P\left[DM \geq dm \mid EDP = edp\right] \tag{1}$$

The fragility function, illustrated in Figure 1-1(a) is expressed by:

$$F_{dm}\left(edp\right) = \Phi\left(\frac{\ln\left(edp/x_m\right)}{\beta}\right) \tag{2}$$

where Φ denotes the standard normal (Gaussian) cumulative distribution function, x_m denotes the median value of the distribution, and β denotes the logarithmic standard deviation. Both x_m and β are established for each component type and damage state using the methods presented in Section 2. The probability that the component is *in* damage state *dm*, given *EDP* = *edp*, is given by

$$
\begin{aligned}
P\left[DM = dm \mid EDP = edp\right] &= 1 - F_1(edp) && dm = 0 \\
&= F_{dm}\left(edp\right) - F_{dm+1}\left(edp\right) && 1 \le dm < N \\
&= F_{dm}\left(edp\right) && dm = N
\end{aligned} \tag{3}
$$

where N denotes the number of possible damage states for the component, in addition to the undamaged state, and $dm = 0$ denotes the undamaged state. Equation 3 is illustrated in Figure 1-1(b) for a component with $N = 3$. Note that, where $N \geq 2$ and $\beta_i \neq \beta_j$ for two damage states $i \neq j$, Equation 3 can produce a meaningless negative probability at some levels of *edp*. This case is addressed in Section 3.4.

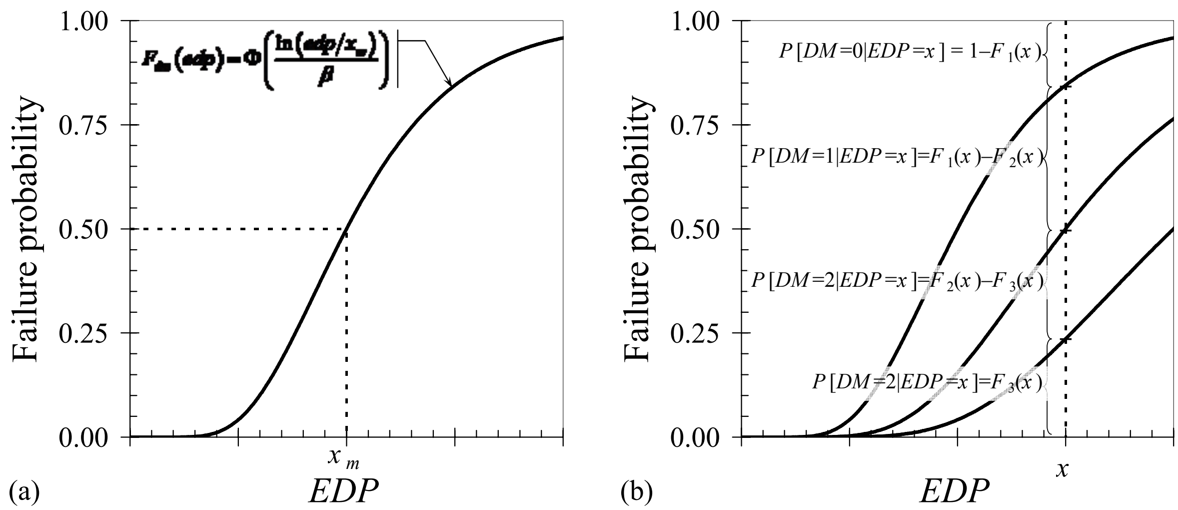

Figure 1-1. Illustration of (a) fragility function, and (b) evaluating damage-state probabilities

SECTION 1 COMMENTARY

C1.1 OBJECTIVE

The text is intended as a resource to create fragility functions from damage data. The main text offers a recipe for fragility functions under each of six situations ranging from the best (known *EDP* at which components failed) to the worst (no data at all). The commentary provides alternative approaches and additional support.

The text provides guidance on developing fragility functions, but the analyst must judge the appropriateness of the data and the method. Application of the equations presented here is not a substitute for understanding the processes that lead to component failure and their implications for fragility.

The math is kept simple: no calculus is required, and the only function that is likely to be unfamiliar to the user is the Gaussian cumulative distribution function and its inverse, both of which are available on spreadsheet software (e.g., normsdist and normsinv, respectively, in Microsoft Excel).

The text ignores *EDP*s that depend on damage to the component, as in a structural member. Ref. [2] suggests that even with structural members, sensitivity of *EDP* to component damage can be ignored without substantial error in the loss analysis.

Commentary section C1.3 discusses alternative forms of the fragility function and shows how to convert from compound lognormal or normal CDF to lognormal.

C1.2 DOCUMENTATION REQUIREMENTS

Well defined EDPs. Only scalar *EDP*s are addressed here. According to Ref [3], equipment damage in earthquakes often results from interaction of adjacent components, which cannot be reflected solely by *EDP*. This issue is ignored or treated implicitly here by assuming or requiring that damage data are collected from specimens that are representative of actual conditions in buildings, with the attendant possibility of interaction.

A mathematical function with all parameters defined. Names for methods A-E and U are selected for mnemonic value, to imply decreasing quality from A to E, and to separate U from the quality ranking.

Table 1-3. Some EDPs in use. The table includes *EDP*s for structural as well as nonstructural components.

C1.3 DEFINITION OF FRAGILITY FUNCTIONS AND PROBABILISTIC DAMAGE STATE

In the present effort, fragility functions are idealized using the lognormal cumulative distribution function, whose parameters x_m and β are listed in Table 1-2 as part of the basic requirements for a fragility function. Most fragility data fit the lognormal distribution better than other distributions. If another functional form better fits the data, then the fragility need not be expressed as lognormal. Examples include the normal distribution, given by Equation 4, where μ and σ are parameters of the distribution fit to the data, the shifted lognormal, given by Equation 5, where c, x_m, and β are parameters of the distribution fit to the data, and the compound lognormal, discussed next.

$$F_{dm}\left(edp\right) = \Phi\left(\frac{edp-\mu}{\sigma}\right) \tag{4}$$

$$F_{dm}\left(edp\right) = \Phi\left(\frac{\ln\left(\left(edp-c\right)/\left(x_m-c\right)\right)}{\beta}\right) \tag{5}$$

The equation for Φ is expressed in [4]. Both Φ and Φ^{-1} are generally built-in functions of spreadsheet and other mathematical software.

Within the nuclear power industry, component fragility functions are sometimes expressed as compound lognormally distributed: x_m is itself treated as lognormally distributed with median value x_{mm} and logarithmic standard deviation β_u. A second term, denoted here by β_r, denotes the logarithmic standard deviation of the fragility function given a particular value of x_m, denoted here by x_m^*. Under the compound lognormal fragility function,

$$P\left[x_m \le x_m^*\right] = \Phi\left(\frac{\ln\left(x_m^*/x_{mm}\right)}{\beta_u}\right)$$
$$P\left[DM \ge dm \mid EDP = edp, x_m = x_m^*\right] = \Phi\left(\frac{\ln\left(edp/x_m^*\right)}{\beta_r}\right) \tag{6}$$

When compound lognormal fragility functions are encountered, one can calculate x_m and β from x_{mm}, β_u, and β_r,

$$\beta = \sqrt{\beta_u^2 + \beta_r^2}$$
$$x_m = x_{mm} \tag{7}$$

and then use x_m and β with Equations 2 and 3.

Converting to lognormal from normal fragility functions. Normal (Gaussian) cumulative distributions functions are sometimes used for fragility functions. When they are encountered, one can calculate x_m and β from the parameters of a Gaussian distribution if necessary. Let μ denote the mean value of the Gaussian fragility function, and let σ denote its standard deviation. Then

$$\beta = \sqrt{\ln\left(1 + \left(\sigma/\mu\right)^2\right)}$$
$$x_m = \mu \Big/ \sqrt{1 + \left(\sigma/\mu\right)^2} \tag{8}$$

2 DERIVING FRAGILITY FUNCTIONS

This section provides mathematical procedures for developing fragility functions.

2.1 METHOD A, ACTUAL *EDP*: ALL SPECIMENS FAILED AT OBSERVED *EDP*

Let

M = number of specimens tested to failure
i = index of specimens, $i \in \{1, 2, \ldots M\}$
r_i = *EDP* at which damage was observed to occur in specimen i.

Then x_m and β are estimated by

$$x_m = \exp\left(\frac{1}{M}\sum_{i=1}^{M}\ln r_i\right)$$
$$\beta = \sqrt{\left(\frac{1}{M-1}\sum_{i=1}^{M}\left(\ln\left(r_i/x_m\right)\right)^2\right)^2 + \beta_u^2} \tag{9}$$

where β_u = 0.25 if any of the following is true, 0 otherwise:

All specimens were in the same configuration (if applicable)
All specimens had the same installation conditions
All specimens experienced the same loading history
$M < 5$

If one or more of the r_i data appear to lie far from the bulk of the data, either above or below, apply the procedure specified in Section 3.2. Finally, test the resulting fragility function using the Lilliefors goodness-of-fit test (Section 3.3). If it passes at the 5% significance level, the fragility function is acceptable.

2.2 METHOD B, BOUNDING *EDP*: SOME SPECIMENS FAILED, PEAK *EDP* IS KNOWN

Here, the data include the maximum *EDP* to which each of *M* specimens was subjected, and knowledge of whether the specimen exceeded the damage state of interest. Some specimens must be damaged. Let

M = number of specimens observed.
i = index of specimens, $i \in \{1, 2, \ldots M\}$
r_i = maximum *EDP* to which specimen *i* was subjected
f_i = failure indicator for specimen *i*
= 1 if specimen *i* failed (reached or exceeded damage state *dm*)
= 0 otherwise
N = number of *EDP* bins

$$N = \left\lfloor \sqrt{M} \right\rfloor \tag{10}$$

where $\lfloor\ \rfloor$ means the largest integer less than or equal to the term inside the brackets
j = index of data bins, $j \in \{1, 2, \ldots N\}$
a_j = lower *EDP* bound of bin *j*

$$a_j = r_{N(j-1)+1} \tag{11}$$

M_j = number of specimens with $a_j \le r < a_{j+1}$

$$\begin{aligned} M_j &= \sum_{i=1}^{M} H\left(r_i - a_j\right) - H\left(r_i - a_{j+1}\right) && j < N \\ &= \sum_{i=1}^{M} H\left(r_i - a_j\right) && j = N \end{aligned} \tag{12}$$

x_j = natural logarithm of the average *r* within bin *j*

$$\begin{aligned} x_j &= \ln\left(\frac{1}{M_j}\sum_{i=1}^{M} r_i\left(H\left(r_i - a_j\right) - H\left(r_i - a_{j+1}\right)\right)\right) && j < N \\ &= \ln\left(\frac{1}{M_j}\sum_{i=1}^{M} r_i H\left(r_i - a_j\right)\right) && j = N \end{aligned} \tag{13}$$

m_j = number of failed specimens in bin *j*, i.e.,

$$\begin{aligned} m_j &= \sum_{i=1}^{M} f_i\left(H\left(r_i - a_j\right) - H\left(r_i - a_{j+1}\right)\right) && j < N \\ &= \sum_{i=1}^{M} f_i H\left(r_i - a_j\right) && j = N \end{aligned} \tag{14}$$

y_j = inverse standard normal distribution of the failed fraction specimens in bin *j*, i.e.,

$$y_j = \Phi^{-1}\left(\frac{m_j + 1}{M_j + 1}\right) \tag{15}$$

where Φ^{-1} denotes the inverse standard normal distribution and

$$H \quad \begin{aligned} &= 1 \text{ if the value in parentheses is positive} \\ &= \tfrac{1}{2} \text{ if the value in parentheses equals zero} \\ &= 0 \text{ if the value in parentheses is negative} \end{aligned} \tag{16}$$

The fragility function parameters x_m and β are determined by fitting a line $\hat{y} = sx + c$ to the data:

$$\begin{aligned} \beta_r &= \frac{1}{s} \\ &= \frac{\sum_{j=1}^{N}\left(x_j - \overline{x}\right)^2}{\sum_{j=1}^{N}\left(x_j - \overline{x}\right)\left(y_j - \overline{y}\right)} \\ x_m &= \exp\left(-c\beta_r\right) \\ &= \exp\left(\overline{x} - \overline{y}\beta_r\right) \\ \beta &= \sqrt{\beta_r^2 + \beta_u^2} \end{aligned} \tag{17}$$

where

$$\begin{aligned} \overline{x} &= \frac{1}{N}\sum_{j=1}^{N} x_j \\ \overline{y} &= \frac{1}{N}\sum_{j=1}^{N} y_j \end{aligned} \tag{18}$$

and β_u = 0.25 if any of the following is true, 0 otherwise:

All specimens were observed to be in the same configuration (if applicable)
All specimens were observed to have the same installation conditions
All specimens experienced the same loading history
$M < 5$

2.3 METHOD C, CAPABLE *EDP*: NO SPECIMENS FAILED, *EDPS* ARE KNOWN

Given no observations of $DM \geq dm$ and M observations of no damage occurrences of $DM \geq dm$, let

r_i = *EDP* experienced by specimen i (i = 1, 2, ... M)
$r_{max} = max_i\{r_i\}$
r_d = minimum *EDP* experienced by any specimen with distress
r_a = the smaller of r_d and $0.7 \cdot r_{max}$
M_A = number of specimens without apparent distress and with $r_i \geq r_a$
M_B = number of specimens at any level of r_i with distress not suggestive of imminent failure
M_C = number of specimens at any level of r_i with distress suggestive of imminent failure
r_m = r_{max} if $M_B + M_C = 0$
= $0.5 \cdot (r_{max} + r_a)$ otherwise
S = subjective failure probability at r_m

$$S = (0.5M_C + 0.1M_B)/(M_A + M_B + M_C) \tag{19}$$

Use Table 2-1 to determine $F_{dm}(r_m)$ and Equation 20 to determine β and x_m.

Table 2-1. Example values of exp(–zβ)

Conditions	$F_{dm}(r_m)$	**Z**	**exp(–zβ), β=0.4**
$M_A \geq 3$ and $S = 0$	0.01	-2.326	2.54
$M_A < 3$ and $S \leq 0.075$	0.05	-1.645	1.93
$0.075 < S \leq 0.15$	0.10	-1.282	1.67
$0.15 < S \leq 0.3$	0.20	-0.842	1.40
$S > 0.3$	0.40	-0.253	1.11

$$\begin{aligned} \beta &= 0.4 \\ z &= \Phi^{-1}\left(F_{dm}\left(r_m\right)\right) \\ x_m &= r_m \exp\left(-z\beta\right) \end{aligned} \tag{20}$$

2.4 METHOD D, DERIVED FRAGILITY FUNCTIONS: ANALYTICAL METHOD

If the capacity of the component can be calculated in terms of *edp* using average material properties and dimensions, and β cannot, then assume a β value and calculate x_m. Let r denote the calculated capacity of the component to resist damage state *dm*, including consideration of any anchorage or bracing. Then

$$\begin{aligned} x_m &= 0.92r \\ \beta &= 0.4 \end{aligned} \tag{21}$$

If β and r can both be calculated by analysis, then use

$$x_m = \frac{r}{\sqrt{\exp\left(\beta^2\right)}} \tag{22}$$

2.5 METHOD E, EXPERT OPINION

Select one or more experts with professional experience in the design or post-earthquake damage observation of the component of interest. Solicit their advice using the format shown in Figure 2-1. Note the suggested inclusion of representative images, which should be recorded with the responses. If an expert refuses to provide estimates or limits them to certain conditions, either narrow the component definition accordingly and iterate, or ignore that expert's response and analyze the remaining ones. Let

N = number of experts providing judgment about a value
i = index of experts, $i \in \{1, 2, \ldots N\}$
x_{mi} = estimated median *EDP* of expert i
x_{li} = estimated lower-bound *EDP* of expert i
w_i = level of expertise of expert i
α = 1.5

$$x_m = \frac{\sum_{i=1}^{N} w_i^{\alpha} x_{mi}}{\sum_{i=1}^{N} w_i^{\alpha}} \tag{23}$$

$$x_l = \frac{\sum_{i=1}^{N} w_i^{\alpha} x_{li}}{\sum_{i=1}^{N} w_i^{\alpha}} \tag{24}$$

$$\beta = \frac{\ln\left(x_m / x_l\right)}{1.28} \tag{25}$$

If Equation 25 produces $\beta < 0.4$, either justify the β, or use Equation 24 and Equation 26:

$$\begin{aligned} \beta &= 0.4 \\ x_m &= 1.67 x_l \end{aligned} \tag{26}$$

Figure 2-1. Form for soliciting expert judgment on component fragility

Objective. This form solicits your judgment about the values of an engineering demand parameter (*EDP*) at which a particular damage state occurs to a particular building component. Judgment is needed because the component may contribute significantly to the future earthquake repair cost, fatality risk, or post-earthquake operability of a building, and because relevant empirical and analytical data are currently impractical to acquire. Your judgment is solicited because you have professional experience in the design or post-earthquake damage observation of the component of interest.

Definitions. Please provide judgment on the damageability of the following component and damage state. Images of a representative sample of the component and damage state may be attached. It is recognized that other *EDP*s may correlate better with damage, but please consider only the one specified here.

Component name: ________________________________

Component definition: ________________________________

____

Damage state name: ________________________________

Damage state definition: ________________________________

____

Relevant *EDP*: ________________________________

Definition of *EDP*: ________________________________

____

Uncertainty; no personal stake. Please provide judgment about this general class of components, not any particular instance, and not one that you personally designed, constructed, checked, or otherwise have any stake in. There is probably no precise threshold level of *EDP* that causes damage, because of variability in design, construction, installation, inspection, age, maintenance, interaction with nearby components, etc. Even if there were such a precise level, nobody might know it with certainty. To account for these uncertainties, **please provide two values of *EDP* at which damage occurs: median and lower bound.**

Estimated median EDP: ____________ *Definition*. Damage would occur at this level of *EDP* in 5 cases out of 10, or in a single instance, you judge there to be an equal chance that your median estimate is too low or too high.

Estimated lower-bound EDP: ____________ *Definition*. Damage would occur at this level of *EDP* in 1 case in 10. In a single case, you judge there to be a 10% chance that your estimate is too high. *Judge the lower bound carefully*. Make an initial guess, then imagine all the conditions that might make the actual threshold *EDP* lower, such as errors in design, construction or installation, substantial deterioration, poor maintenance, more interaction with nearby components, etc. Revise accordingly and record your revised estimate. Research shows that without careful thought, expert judgment of the lower bound tends to be too close to the median estimate, so think twice and do not be afraid of showing uncertainty.

On a 1-to-5 scale, please judge your expertise with this component and damage state, where 1 means "no experience or expertise" and 5 means "very familiar or highly experienced."

Your level of expertise: ____________

Your name: ________________________________ Date: ________________

2.6 METHOD U, UPDATING A FRAGILITY FUNCTION WITH NEW METHOD-B DATA

Here, the data include a preexisting fragility function and a set of M specimens with known maximum excitation and damage state. It is not necessary that any of the specimens experienced damage. (See the commentary for the case of Method-A data, i.e., specimens tested to failure.) Let

M	= number of specimens observed.
i	= index of specimens, $i \in \{1, 2, \ldots M\}$
r_i	= maximum *EDP* to which specimen i was subjected
f_i	= failure indicator for specimen i
	= 1 if specimen i failed (reached or exceeded damage state *dm*)
	= 0 otherwise
x_m	= median from pre-existing fragility function
β	= logarithmic standard deviation from pre-existing fragility function
x'_m	= median of the revised fragility function
β'	= logarithmic standard deviation of the revised fragility function

Calculate the revised median and logarithmic standard deviation as follows:

$$x'_m = \exp\left(\sum_{j=1}^{5} w'_j \ln\left(x_{mj}\right)\right)$$
$$\beta' = \sum_{j=1}^{5} w'_j \beta_j \qquad (27)$$

where

$$w'_j = \frac{w_j \prod_{i=1}^{M} L(i,j)}{\sum_{j=1}^{5} w_j \prod_{i=1}^{M} L(i,j)} \qquad (28)$$

$$L(i,j) = 1 - \Phi\left(\frac{\ln\left(r_i / x_{mj}\right)}{0.707\beta_j}\right) \qquad if\ f_i = 0$$
$$= \Phi\left(\frac{\ln\left(r_i / x_{mj}\right)}{0.707\beta_j}\right) \qquad if\ f_i = 1 \qquad (29)$$

where Π denotes the product of the terms that come after it, and where

$$\begin{aligned}
&x_{m1} = x_{m4} = x_{m5} = x_m \\
&x_{m2} = x_m e^{-1.22\beta} \\
&x_{m3} = x_m e^{1.22\beta} \\
&\beta_1 = \beta_2 = \beta_3 = \beta \\
&\beta_4 = 0.64\beta \\
&\beta_5 = 1.36\beta \\
&w_1 = 1/3 \\
&w_2 = w_3 = w_4 = w_5 = 1/6
\end{aligned} \qquad (30)$$

SECTION 2 COMMENTARY

C2.1 METHOD A, ACTUAL *EDP*: ALL SPECIMENS FAILED AT OBSERVED *EDP*

These are the most informative data for creating fragility functions. They are most common where a *DM* can be associated with a point on the observed force-deformation behavior of a component, such as a yield point. Alternatively, specimens are subjected to increasing levels of *EDP*. The test is interrupted after each level of *EDP* is imposed, and the specimen examined for damage. The justification for the β_u term is that uniform configuration, installation conditions, loading history, and small sample size are likely to result in an underestimate of β.

C2.2 METHOD B, BOUNDING *EDP*: SOME SPECIMENS FAILED, PEAK *EDP* IS KNOWN

Here, the data include the maximum *EDP* to which each of *M* specimens was subjected, and knowledge of whether the specimen exceeded the damage state of interest. The method works best for cases where $M \geq 25$ (an alternative is presented later). Data must not be biased by damage state, i.e., specimens must not be selected because they experienced damage. The data are grouped into bins by ranges of *EDP*, where each bin has approximately the same number of specimens in it. For each bin, one calculates the fraction of specimens that failed and the bin-average *EDP*. These serve as independent data points of failure probability and *EDP*. The choice of $N \approx M^{1/2}$ is recommended as it creates a number of bins approximately equal to the number of specimens in each bin.

The approach discussed in the text converts Equation 2 to a linear regression problem by taking the inverse Gaussian CDF of each side and fitting a line $\hat{y} = sx + c$ to the data, as illustrated in Figure 2-2. In Equation 15, 1 is added to numerator and denominator to deal with cases with zero failures in the bin.

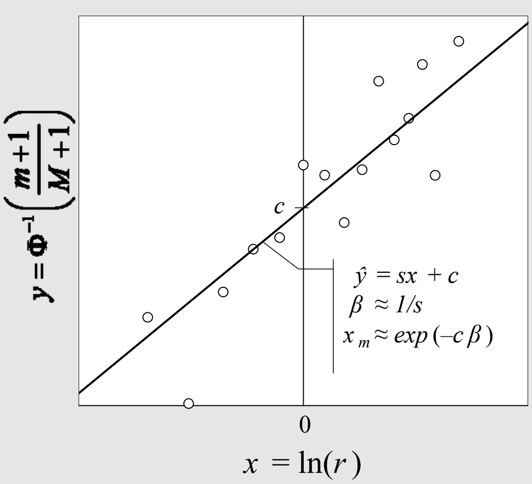

Figure 2-2. Estimating lognormal fragility function parameters from binned failure data with peak *EDP*

As with Method A, the justification for the β_u term is that uniform configuration, installation conditions, loading history, and small sample size are likely to result in an underestimate of β.

Method B-2. Another way to fit a fragility function to bounding-*EDP* data is to perform a least-squares fit to the binary failure data, as illustrated in **Figure 2-3**. Let

M = number of specimens observed.

i = index of specimens, $i \in \{1, 2, \ldots M\}$
r_i = EDP to which specimen i was subjected
f_i = failure indicator for specimen i
= 1 if specimen i failed (reached or exceeded damage state dm)
= 0 otherwise
Φ = standard normal (Gaussian) cumulative distribution function

Find x_m, β_r to minimize ε^2 such that:

$$\varepsilon^2 = \frac{1}{M}\sum_{i=1}^{M}\left(f_i - \Phi\left(\frac{\ln\left(r_i/x_m\right)}{\beta_r}\right)\right)^2$$
$$x_m > 0 \qquad (31)$$
$$\beta_r \geq 0.2$$

and calculate β:

$$\beta = \sqrt{\beta_r^2 + \beta_u^2} \qquad (32)$$

where β_u = 0.25 if any of the following is true, 0 otherwise:

All specimens were observed to be in the same configuration (if applicable)
All specimens were observed to have the same installation conditions
All specimens experienced the same loading history
$M < 5$

Spreadsheet and other mathematical software have built-in solver routines to perform this calculation. The advantage of this approach is that it avoids errors associated with bin-average *EDP*s.

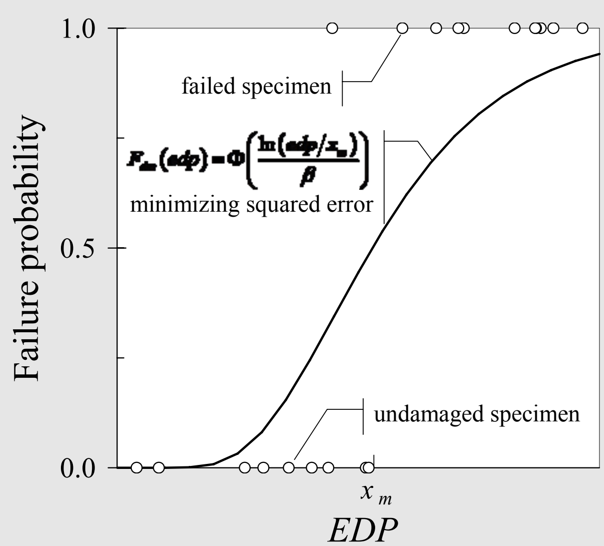

Figure 2-3. Lognormal fragility function from binary failure data with peak *EDP* by least-squares fit

This application of least-squares parameter estimation for fragility functions using binary data is introduced here, and was tested by simulation.

Method B-3. For bins with varying number of specimens, one can perform a weighted least-squares fit to failure-rate data, using the number of specimens in each bin as the bin weight. Let

M = number of specimens observed
i = index of specimens, $i \in \{1, 2, \ldots M\}$
r_i = maximum *EDP* to which specimen i was subjected
f_i = failure indicator for specimen i
= 1 if specimen i failed (reached or exceeded damage state *dm*)
= 0 otherwise
N = number of *EDP* bins, selected by the analyst
j = index of data bins, $j \in \{1, 2, \ldots N\}$
a_j = lower *EDP* bound of bin j, selected by the analyst
M_j = number of specimens in bin j

$$\begin{aligned} M_j &= \sum_{i=1}^{M} H\left(r_i - a_j\right) - H\left(r_i - a_{j+1}\right) && j < N \\ &= \sum_{i=1}^{M} H\left(r_i - a_j\right) && j = N \end{aligned} \tag{33}$$

x_j = average r within bin j

$$\begin{aligned} x_j &= \frac{1}{M_j}\sum_{i=1}^{M} r_i\left(H\left(r_i - a_j\right) - H\left(r_i - a_{j+1}\right)\right) && j < N \\ &= \frac{1}{M_j}\sum_{i=1}^{M} r_i H\left(r_i - a_j\right) && j = N \end{aligned} \tag{34}$$

m_j = number of failed specimens in bin j

$$\begin{aligned} m_j &= \sum_{i=1}^{M} f_i\left(H\left(r_i - a_j\right) - H\left(r_i - a_{j+1}\right)\right) && j < N \\ &= \sum_{i=1}^{M} f_i H\left(r_i - a_j\right) && j = N \end{aligned} \tag{35}$$

y_j = failure rate in bin j

$$y_j = \frac{m_j}{M_j} \tag{36}$$

where

H = 1 if the value in parentheses is positive
= ½ if the value in parentheses equals zero (37)
= 0 if the value in parentheses is negative

Find x_m and β_r to minimize ε^2 such that:

$$\begin{aligned} \varepsilon^2 &= \frac{1}{M}\sum_{j=1}^{M} M_j\left(y_j - \Phi\left(\frac{\ln\left(x_j / x_m\right)}{\beta_r}\right)\right)^2 \\ x_m &> 0 \\ \beta_r &\geq 0.2 \end{aligned} \tag{38}$$

Calculate β:

$$\beta = \sqrt{\beta_r^2 + \beta_u^2} \tag{39}$$

where β_u = 0.25 if any of the following is true, 0 otherwise:

All specimens were observed to be in the same configuration (if applicable)
All specimens were observed to have the same installation conditions
All specimens experienced the same loading history
$M < 5$

Again, spreadsheet and other mathematical software have built-in solver routines to perform this calculation. The advantage of this approach is that bins can have varying number of specimens. One useful application is earthquake experience data where varying numbers of specimens are observed at each of several facilities, and the *EDP* for each bin is taken as a single estimate for the facility.

C2.3 METHOD C, CAPABLE *EDP*: NO SPECIMENS FAILED, *EDPS* ARE KNOWN

Method C is introduced in the present guidelines. It is based on and expanded from Ref [5]. It addresses the best case for this type of data, i.e., many specimens, none of which had apparent distress, and several of which were subjected to *EDP* near the maximum value. It also addresses the more general case, including situations where few specimens experienced *EDP* near the maximum, or where some specimens experienced distress short of the damage state of interest, or both.

The procedure creates a bin-average subjective failure probability S for a bin of specimens at the high end of the tested range of *EDP*, and assigns a response value to this bin of specimens. The bin includes all specimens with some distress, the lowest of which has $EDP = r_d$, and all specimens without distress that were subjected to *EDP* of at least r_d or 0.7 times the largest level of *EDP* to which any specimen was subjected. The specimens in this bin without apparent distress are assigned 0% subjective failure probability, 10% for specimens with distress not suggestive of imminent failure, and 50% for specimens with distress suggestive of imminent failure. It assigns to this bin the median *EDP* of all the specimens in the bin, denoted by r_m. Combining the point on the fragility function (r_m, S) with an assumed $\beta = 0.4$ produces a fragility function consistent with the assigned subjective failure probabilities. The precise interpretation of "distress suggestive of imminent failure" is left to the analyst.

The value of $\beta = 0.4$ is selected by the judgment of the ATC-58 Nonstructural Products team as an approximate median of observed β values. The use of $\beta = 0.3$ produces a more conservative estimate, if conservativeness is desired.

C2.4 METHOD D, DERIVED FRAGILITY FUNCTIONS: ANALYTICAL METHOD

The capacity of some components can be calculated by modeling the component as a structural system, and determining the level of excitation (e.g., acceleration or shear deformation) that would cause the system to reach *dm*. Other components may be amenable to fault tree analysis; see, e.g., [6].

Equation 21 assumes that $\beta = 0.4$ and calculates the median of a lognormal distribution from the mean value and β.

C2.5 METHOD E, EXPERT OPINION

There are several methods for eliciting expert opinion, from the completely ad-hoc to structured processes involving multiple experts, self-judgment of expertise, and iteration to examine major discrepancies between experts. To elicit expert opinion on probabilistic quantities properly requires attention to clear definitions, biases, assumptions, and expert qualifications. The method proposed here employs [7] for probability encoding and [8] for expert qualification, simplified for practicality. Other relevant references include [9, 10, 11, and 12].

Ref [7] offers methods for eliciting expert judgment where the expert provides probabilities associated with specified values of the parameter of interest (P-method); values for specified probabilities (V-method); and both values and probabilities with neither fixed (PV-method). The method presented here employs a V-method, judging it to be the simplest and most intuitive of the three, and drawing on the precedent of [12]. Ref [7] also provides for two modes of response: direct, where the expert provides numbers as a response; and indirect, where the expert chooses between two or more bets or alternatives that are adjusted until the expert is indifferent between the alternatives, from which values or probabilities can be extracted. Direct response is selected here as less time-consuming and requiring less calculation. Ref [7] provides for soliciting many points on a probability distribution, using a mechanical device to depict probability visually, and fitting a curve to these data. These refinements are rejected as too cumbersome for present purposes. Nonetheless, the same basic steps are employed here as proposed in [7]:

1. Motivating, in which the purpose and importance of the effort are explained.
2. Structuring, in which one defines the quantities to be judged.
3. Conditioning, in which one specifies the conditions on which the quantities are to be judged.
4. Encoding, in which the expert expresses judgments in quantitative probabilistic terms.
5. Verifying, in which the analyst checks the values for consistency.

Ref [9] represents the classic text on soliciting expert opinion via the Delphi process. Ref [8] demonstrates that if one employs the Delphi process and also solicits judgment from respondents about their own level of expertise, the results tend to be more accurate if one screens the judgments to include only those who rate their expertise highly. The authors show that self-rating improves estimates more than feedback, i.e., iteration to discuss discrepancies. Expertise was self-evaluated on a 1-to-5 scale, and group error compared with average group self-rating. The data suggest that those who rate their expertise as 1 estimate quantitative values with three times greater error than the most-expert.

Ref [12] modified the screening approach to include all experts, but to create a weighted average of the solicited judgment, where the weight is based on the experts' self-judgment of their expertise and confidence about the particular value they were estimating. This seems to be a reasonable compromise and is employed here. Its authors used a 1-10 scale for expertise, and weighted response by expertise raised to the 4^{th} power, so that the most-expert respondent would have his or her response weighted 10,000 times greater than the least-expert respondent. Using a 1-to-5 scale would make an equivalent α value of 6, which seems to be too high in light of [8]. Using a $\alpha = 1.5$, an expert with $w = 5$ is weighted twice as heavily as one with $w = 3$, and 11 times the weight of an expert with $w = 1$.

Ref [10] supports the notion that Delphi estimation is an orderly process and provides a reasonable means for obtaining group consensus on a forecast, which lends weight to the notion that a similar process can reasonably be applied here. Ref [11] finds that the distribution of expert opinion on uncertain variables that must have a positive value (in that work, future dates) tends to be lognormally distributed, which supports the use of Equation 25.

Ref [12] employed an iterative process of judging values, with multiple rounds of judgment followed by discussion of any large discrepancies between experts. Such an iterative approach might be practical in some circumstances, but perhaps not in general. For simplicity, no iteration is proposed here.

Refs [13] and [14] support the notion that by establishing the *EDP* at which the component has 10% failure probability, the overall reliability of the component is fairly insensitive to β, hence the value of directly encoding experts' judgment of this value in particular.

Regarding Equation 26, it is common for experts to express overconfidence in an uncertain variable, such as the *EDP* at which damage will occur. If, despite the advice in Figure 2-1 regarding uncertainty, the results of the survey produce $\beta < 0.4$, and this low value of β cannot be justified, use the judged x_l to anchor the fragility function, apply $\beta = 0.4$, and calculate the resulting value of x_m.

C2.6 METHOD U, UPDATING A FRAGILITY FUNCTION WITH NEW METHOD-B DATA

This is the most complicated method presented here. Yet it still requires of the analyst no more mathematics than calculating sums, products, and cumulative Gaussian distribution functions. It uses Bayesian updating to revise the parameters x_m and β of a preexisting fragility function, based on observation of a set of M specimens whose *EDP* and damage state pairs (r, f) have been observed, as after an earthquake.

To perform the updating, it is recognized that x_m and β are themselves uncertain, and have some probability distributions of their own. It is these distributions that will be revised, and new expected values for x_m and β calculated. The prior probability distribution of x_m is taken as lognormal with median equal to the x_m value in the preexisting fragility function, and logarithmic standard deviation taken as 0.707 times the β of the preexisting fragility function. This is consistent with a compound lognormal fragility function and the assumption that $\beta_r = \beta_u = 0.707\beta$.

The prior probability distribution of β is taken as normal with expected value equal to the β of the preexisting fragility function, and coefficient of variation (COV) taken to be 0.21. This COV is selected because it provides for 98% probability that β is within the bounds of 0.5 and 1.5 times the prior β, which agrees with the general range for β of 0.2 to 0.6.

The distributions of x_m and β are assumed to be independent. Their joint distribution is approximated by five discrete points (x_{mj}, β_j), each with probability-like weight w_j (where $j = 1, 2, \ldots 5$). Using a method described in [15], the values of x_{mj}, β_j, and w_j are chosen so that the first five moments of the discrete joint distribution match those of the continuous joint distribution. The principle is illustrated in Figure 2-4, which shows a probability density function of two variables x_m and β (the surface) and the discrete points (bars), each with an associated weight. The first few moments of the points (the mean, variance, etc.) match those of the surface.

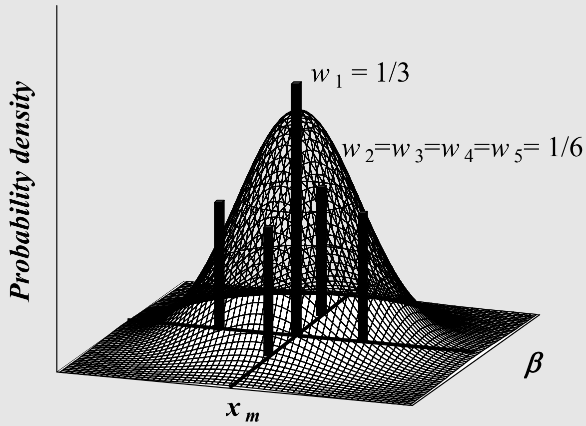

Figure 2-4. Substituting a sample of 5 points (bars) for a continuous joint distribution (surface)

In Equation 28, the weights of these five points are updated to account for the observations, and the revised x_m and β are calculated in Equation 27 based on the updated weights.

Another situation where Bayesian updating can be used is where the data include a preexisting fragility function and a set of M specimens tested to failure, where the *EDP* at which each specimen exceeded the damage state of interest is known. The procedure is the same as with method U, except that r_i denotes the *EDP* at which damage was observed to occur in specimen i, and Equation 29 is replaced by

$$L(i,j) = \phi\left(\frac{\ln\left(r_i/x_{mj}\right)}{0.707\beta_j}\right) \tag{40}$$

where ϕ denotes the standard normal (Gaussian) probability *density* function. Note that ϕ is the bell-shaped curve that is the first derivative of the S-shaped cumulative distribution function, Φ. It can be found in common spreadsheet software, e.g., normdist in MS Excel. Be careful to set software flags to have the function calculate probability density rather than the cumulative distribution.

3 ASSESSING FRAGILITY FUNCTION QUALITY

The previous section provided mathematical procedures for developing fragility functions. Issues associated with the quality of those fragility functions are now addressed, particularly the treatment of competing *EDP*s, goodness-of-fit testing, dealing with fragility functions that cross, and how to assign an overall quality level to a fragility function.

3.1 CONSIDERING COMPETING EDPS

It may be that one is uncertain of which *EDP* mostly matters to component damage. Create fragility functions for each possibly relevant *EDP*. If different *EDP*s have the same coefficient of variation, choose the fragility function that has the lowest β. If different *EDP*s have differing coefficient of variation, let

N = number of competing *EDP*s being considered

i = index of *EDP*s, i.e., $i \in \{1, 2, \ldots N\}$. For example, *EDP* 1 might be peak diaphragm acceleration, *EDP* 2 might be floor spectral acceleration response, etc.

$\beta_{EDPi|S}$ = residual logarithmic standard deviation of *EDP i* given the structural model *S*, assumed to be independent of *IM*

$\beta_{DM|EDPi}$ = logarithmic standard deviation of *EDP i* causing $DM \geq dm$ to be exceeded, given *EDP*, assumed to be independent of *S* and *IM*. This is "the β" of the fragility function created in terms of *EDP i*.

$\beta_{DM,EDPi|S}$ = logarithmic standard deviation of *EDP i* causing $DM \geq dm$ to be exceeded, given *S*, assumed to be independent of *IM*:

$$\beta_{DM,EDPi|S} = \sqrt{\beta^2_{EDPi|S} + \beta^2_{DM|EDPi}} \tag{41}$$

Choose the fragility function that has the lowest value of $\beta_{DM,EDPi|S}$.

3.2 DEALING WITH OUTLIERS BY PEIRCE'S CRITERION

It is possible that one or more samples r_i of Method-A data are spurious, and reflect experimental errors rather than the true *EDP*s at which the specimens failed. In cases where these r_i lie far from the bulk of the data, investigate whether the data reflect real issues in the damage process that may recur, especially where $r_i << x_m$. If there is no indication that these data reflect a real recurring issue in the damage process, apply the following procedure (Peirce's criterion) to test and eliminate doubtful observations of r_i.

1. Calculate $\ln(x_m)$ and β of the complete data set.
2. Let *D* denote the number of doubtful observations, and let *R* denote the maximum distance of an observation from the body of the data, defined as:

$$R = \frac{\left|\ln(r) - \ln(x_m)\right|_{\max}}{\beta} \tag{42}$$

   where x_m, β, and *M* are as defined for Equation 9, *r* is a measured *EDP* value, and *R* is shown in Table 3-1. Assume $D = 1$ first, even if there appears to be more than one doubtful observation.
3. Calculate the maximum allowable deviation: $|\ln(r) - \ln(x_m)|_{\max}$. Note that this can include $r >> x_m$ and $r << x_m$.
4. As with Equation 9, let r_i denote the *EDP* at which damage was observed to occur in specimen *i*. For any suspicious measurement r_i, obtain $|\ln(r_i) - \ln(x_m)|$.
5. Eliminate the suspicious measurements if:

$$\left|\ln(r_i) - \ln(x_m)\right| > \left|\ln(r) - \ln(x_m)\right|_{\max} \tag{43}$$

6. If this results in the rejection of one measurement, assume D=2, keeping the original values of x_m, β, and *M*, and go to step 8.
7. If more than one measurement is rejected in the above test, assume the next highest value of doubtful observations. For example, if two measurements are rejected in step 5, assume the case of $D = 3$, keeping the original values of x_m, β, and *M* as the process is continued.
8. Repeat steps 2 – 5, sequentially increasing *D* until no more data measurements are eliminated.
9. Obtain x_m and β of the reduced data set by Equation 9.

Table 3-1. Parameters for applying Peirce's criterion

M	*D*=1	*D*=2	*D*=3	*D*=4	*D*=5	*D*=6	*D*=7	*D*=8	*D*=9
3	1.1960								
4	1.3830	1.0780							
5	1.5090	1.2000							
6	1.6100	1.2990	1.0990						
7	1.6930	1.3820	1.1870	1.0220					
8	1.7630	1.4530	1.2610	1.1090					
9	1.8240	1.5150	1.3240	1.1780	1.0450				
10	1.8780	1.5700	1.3800	1.2370	1.1140				
11	1.9250	1.6190	1.4300	1.2890	1.1720	1.0590			
12	1.9690	1.6630	1.4750	1.3360	1.2210	1.1180	1.0090		
13	2.0070	1.7040	1.5160	1.3790	1.2660	1.1670	1.0700		
14	2.0430	1.7410	1.5540	1.4170	1.3070	1.2100	1.1200	1.0260	
15	2.0760	1.7750	1.5890	1.4530	1.3440	1.2490	1.1640	1.0780	
16	2.1060	1.8070	1.6220	1.4860	1.3780	1.2850	1.2020	1.1220	1.0390
17	2.1340	1.8360	1.6520	1.5170	1.4090	1.3180	1.2370	1.1610	1.0840
18	2.1610	1.8640	1.6800	1.5460	1.4380	1.3480	1.2680	1.1950	1.1230
19	2.1850	1.8900	1.7070	1.5730	1.4660	1.3770	1.2980	1.2260	1.1580
20	2.2090	1.9140	1.7320	1.5990	1.4920	1.4040	1.3260	1.2550	1.1900
>20	$a \ln M + b$								
a	0.4094	0.4393	0.4565	0.4680	0.4770	0.4842	0.4905	0.4973	0.5046
b	0.9910	0.6069	0.3725	0.2036	0.0701	-0.0401	-0.1358	-0.2242	-0.3079

3.3 GOODNESS OF FIT TESTING

Test fragility functions created using Method A for goodness of fit. Calculate

$$D = \max_X \left| F_{dm}(edp) - S_M(edp) \right| \tag{44}$$

where $S_M(edp)$ denotes the sample cumulative distribution function

$$S_M(edp) = \frac{1}{M} \sum_{i=1}^{M} H(r_i - edp) \tag{45}$$

and H is given by Equation 16. If $D > D_{\mathrm{crit}}$ from Table 3-2, the fragility function fails the goodness of fit test. The result is used in assigning a quality level to the fragility function. Use $\alpha = 0.05$.

Table 3-2. Critical values for the Lilliefors Test

Significance Level	D_{crit}
$\alpha = 0.15$	$0.775 / (M^{0.5} - 0.01 + 0.85M^{-0.5})$
$\alpha = 0.10$	$0.819 / (M^{0.5} - 0.01 + 0.85M^{-0.5})$
$\alpha = 0.05$	$0.895 / (M^{0.5} - 0.01 + 0.85M^{-0.5})$
$\alpha = 0.025$	$0.995 / (M^{0.5} - 0.01 + 0.85M^{-0.5})$

3.4 FRAGILITY FUNCTIONS THAT CROSS

Some components have two or more possible damage states, with a fragility function for each. For any two (cumulative lognormal) fragility functions i and j with medians $x_{mj} > x_{mi}$ and logarithmic standard deviations $\beta_i \neq \beta_j$, the fragility functions will cross. In such a case, either replace Equation 2 with

$$F_i\left(edp\right) = \max_j \left\{ \Phi\left(\frac{\ln\left(edp/x_{mj}\right)}{\beta_j} \right) \right\} \qquad \text{for all } j \geq i \tag{46}$$

or establish x_m and β values for the various damage states independently, and then revise them as:

$$\begin{aligned} \beta_i' &= \frac{1}{N}\sum_{i=1}^{N}\beta_i \\ x_{mi}' &= \exp\left(1.28\left(\beta' - \beta_i\right) + \ln x_{mi}\right) \end{aligned} \qquad \text{for all } i \tag{47}$$

where i indexes the damage state and N is the number of damage states in addition to undamaged.

3.5 ASSIGNING A SINGLE QUALITY LEVEL TO A FRAGILITY FUNCTION

Assign a fragility function a quality level of high, medium, or low, as shown in Table 3-3. Report the quality of fragility functions used with the loss estimate.

Table 3-3. Fragility function quality level

Quality	**Method**	**Peer reviewed***	**Number of specimens**	**Other**
High	A	Yes	≥ 5	Passes Lilliefors test at 5% significance level. Examine and justify (a) differences of greater than 20% in x_m or β, compared with past estimates, and (b) any case of $\beta < 0.2$ or $\beta > 0.6$.
	B	Yes	≥ 20	Examine and justify (a) differences of greater than 20% in x_m or β, compared with past estimates, and (b) any case of $\beta < 0.2$ or $\beta > 0.6$.
	U	Yes	≥ 6	Prior was at least moderate quality
Moderate	A		≥ 3	Examine and justify any case of $\beta < 0.2$ or $\beta > 0.6$.
	B		≥ 16	Examine and justify any case of $\beta < 0.2$ or $\beta > 0.6$.
	C	Yes	≥ 6	
	D	Yes		
	E	Yes		At least 3 experts with $w \geq 3$
	U		≥ 6	or prior was moderate quality
Low				All other cases

* Data and derivation published in a peer-reviewed archival journal.

SECTION 3 COMMENTARY

C3.1 CONSIDERING COMPETING EDPS

If competing EDPs themselves have differing degrees of uncertainty for the same structural model, *EDP* uncertainty must also be considered. In such a case, it is assumed that each *EDP* is lognormally distributed conditioned on the structural model *S* and on *IM*, and that the fragility functions being examined are in the form of cumulative lognormal distributions of *EDP* at which the damage state *DM* is reached or exceeded. Then selecting the fragility function that uses *EDP* with the lowest value of $\beta_{DM,EDPi|S}$ will produce the lowest uncertainty in damage state, after accounting for uncertainty in *EDP*.

An additional refinement to Equation 41 would be to account for the fact that β_{Si} might vary with *EDP*, and that lower values are *EDP* are more likely that higher ones in a single event. Such a case is not considered here.

C3.2 DEALING WITH OUTLIERS BY PEIRCE'S CRITERION

Do not assume that Method-A data that lie far from other specimens necessarily result from measurement or other procedural error. To the extent possible, investigate whether outliers might reflect an anomalous, possibly infrequent, but real causal pattern in the damage process. This is particularly important for cases where $r_i << x_m$, and failure might affect life safety. Such outliers could be a signal that a better damage model is needed. In any event, outliers should be reported along with the rest of the data used in the analysis, even if the outliers are removed.

If available evidence suggests that the outliers reflect measurement or other procedural errors, there are several procedures for treating them. Ref [16] summarizes and compares Chauvenet's criterion and Peirce's criterion. Chauvenet's criterion is simpler and more commonly used, but relies on an arbitrary assumption and does not distinguish between cases of a single outlier and many. Peirce's criterion is recommended here. The procedure presented here is adapted from Ref [16] for lognormally distributed data. Peirce's criterion is based on the principle that " …the proposed observations should be rejected when the probability of the system of errors [Ref 16 interprets "the system of errors" as the actual deviations from the mean] obtained by retaining them is less than that of the system of errors obtained by their rejection multiplied by the probability of making so many, and no more, abnormal observations." In Table 3-1, the equation for *R* for $20 < M \leq 60$ was fit to the data in Ref [16], and is introduced here for the sake of brevity.

C3.3 GOODNESS OF FIT TESTING

Goodness-of-fit testing refers to mathematical tests to check that assumed distribution adequately fits the data. Common tests include the chi-square (χ^2), Kolmogorov-Smirnov (K-S), and Lilliefors tests [17], the last of which is proposed for use here. The Lilliefors test is a special case of the K-S test, applicable for testing whether a set of observations adequately fits a normal or lognormal distribution, when the mean and variance are estimated from the sample, as is the case in the present application. If the fragility function fails the goodness of fit test, one can reject the hypothesis that the observations are from a normal population. It is common to use the 5% significance level. The equations in Table 3-2 are shown here for brevity, rather than using the longer table from Ref [17]; they are taken from [18] and were checked for accuracy.

C3.4 FRAGILITY FUNCTIONS THAT CROSS

It can be shown that the failure probability for damage state $j > i$ will exceed that of damage state *i* under the following conditions:

$$edp < \exp\left(\frac{\beta_j \ln x_{mi} - \beta_i \ln x_{mj}}{\beta_j - \beta_i}\right): \quad \beta_i < \beta_j$$
$$edp > \exp\left(\frac{\beta_j \ln x_{mi} - \beta_i \ln x_{mj}}{\beta_j - \beta_i}\right): \quad \beta_i > \beta_j \tag{48}$$

producing a negative probability of being in damage state i (Equation 3b). There is no meaning to a negative probability, so fragility functions that cross pose a problem. Figure 3-1(left) illustrates both cases of Equation 48: Comparing F_1 and F_2, the higher damage state has higher β. Comparing F_2 and F_3, the higher damage state has lower β. Figure 3-1(center) illustrates the use of Equation 46: take the fragility function for damage state i as the maximum of all fragility functions $j \geq i$. Figure 3-1(right) illustrates the use of Equation 47: adjust the fragility functions so that they do not cross, and so that they match the originals at the 10% failure probability.

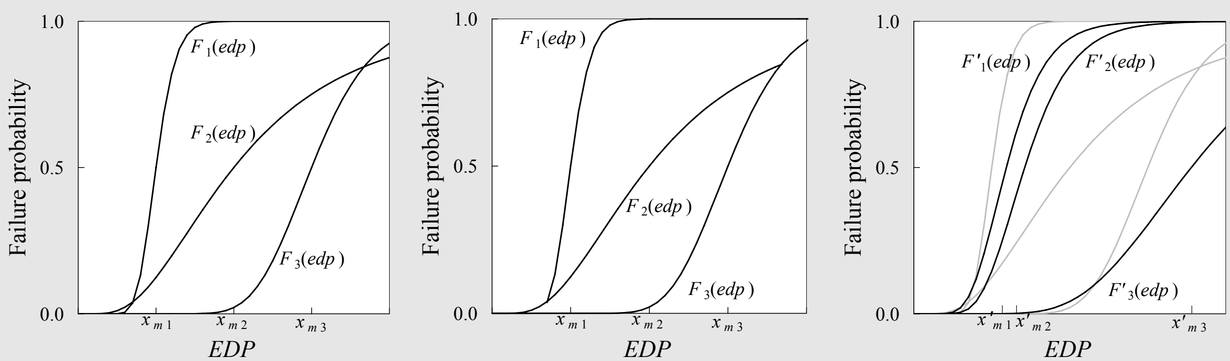

Figure 3-1. Illustration of fragility functions that cross (left); solution with Equation 46 (center) and 47 (right)

See Section C2.5 for support for matching the 10th percentile of capacity, between original and adjusted fragility functions.

C3.5 ASSIGNING A SINGLE QUALITY LEVEL TO A FRAGILITY FUNCTION

It has been observed that system performance is more sensitive to failure rates at low values of *EDP* than at high values. One can perform an additional check of the goodness of fit for a fragility function produced by Methods A and B. Let us refer to this fragility function as the initial fit. Consider the subset of data with $r \leq x_m$ (where x_m is estimated from the initial fit). Fit a fragility function to the subset (referred to here as the subset fragility function). Let x'_m and β' denote the parameters of the subset fragility function. If the quality of the subset fragility function is at least equal to that of the initial fit, and if either of the following conditions is true, then the subset fragility function is preferred to the initial fit.

$$\frac{|x'_m - x_m|}{x_m} \geq 0.20$$
$$\frac{|\beta' - \beta|}{\beta} \geq 0.20 \tag{49}$$

4 EXAMPLES

This section presents examples of each method presented in Section 2, Deriving Fragility Functions.

Example 1. Method A. Ref [19] provides a table of peak transient drift ratios at which 43 specimens of pre-1976 reinforced concrete slab-column connections experienced cracking of no more than 0.3mm width. The data are repeated in Table 4-1 with original specimen numbers. Calculate the fragility function and test goodness of fit.

Solution. The data are sorted in order of increasing r, an index i is added, the statistics $\ln(r_i)$ and $\ln(r_i/x_m)^2$ calculated and summed, as shown in Table 4-2. Using Equation 9,

$$\begin{aligned} x_m &= \exp(1/M \cdot \Sigma_i(\ln(r_i)) \\ &= \exp(-41.6/43) \\ &= 0.38 \end{aligned}$$

$$\begin{aligned} \beta &= (\Sigma_i(\ln(r_i/x_m)^2))/(M-1))^{0.5} \\ &= (6.399/42)^{0.5} \\ &= 0.39 \end{aligned}$$

In Table 4-3, the sample cumulative distribution function is calculated. Note the dummy specimens are added after increases in r, to produce the stepped sample cumulative distribution function shown in Figure 4-1, which is required to comply with Equation 45. Unneeded specimens are omitted from Table 4-3 for brevity.

$$S(r_i) = \frac{i}{M}$$

The fragility function and D are calculated:

$$\begin{aligned} F_{dm}(r_i) &= \Phi\left(\frac{\ln(r_i/x_m)}{\beta}\right) \\ &= \Phi\left(\frac{\ln(r_i/0.38)}{0.39}\right) \end{aligned}$$

$$D = \max_i\left(\left|F_{dm}(r_i) - S_M(r_i)\right|\right)$$

As shown in Table 4-3, D = 0.11. The critical value for the Lilliefors test at the 5% significance level, from Table 3-2, is

$$\begin{aligned} D_{crit} &= 0.895 / (M^{0.5} - 0.01 + 0.85M^{-0.5}) \\ &= 0.895/(43^{0.5} - 0.01 + 0.85 \cdot 43^{-0.5}) \\ &= 0.134 \end{aligned}$$

Since $D < D_{crit}$, the fragility function passes the goodness of fit test.

Table 4-1. Example 1 slab-column connection damage data; r = peak transient, %

Specimen	*r*	Specimen	*r*	Specimen	*r*
3	0.43	21	0.31	69	0.50
4	0.30	22	0.40	71	0.19
5	0.28	23	0.36	72	0.19
6	0.65	24	0.28	73	0.28
7	0.22	25	0.20	74	0.28
8	0.32	26	0.50	75	0.40
9	0.43	27	0.25	76	0.74
10	0.42	28	0.50	77	0.54
11	0.28	59	0.64	78	0.43
12	0.35	60	0.54	79	0.71
16	0.31	61	0.40	80	0.43
17	0.31	62	0.80	81	0.50
18	0.28	66	0.50	82	0.70
19	0.22	67	0.50		
20	0.22	68	0.50		

(Specimens without PTD data are omitted)

Table 4-2. Example 1 calculation of fragility function parameters

i	r_i	$\ln(r_i)$	$\ln(r_i/x_m)^2$	i	r_i	$\ln(r_i)$	$\ln(r_i/x_m)^2$
1	0.19	-1.66	0.481	23	0.40	-0.92	0.003
2	0.19	-1.66	0.481	24	0.42	-0.87	0.010
3	0.20	-1.61	0.412	25	0.43	-0.84	0.015
4	0.22	-1.51	0.299	26	0.43	-0.84	0.015
5	0.22	-1.51	0.299	27	0.43	-0.84	0.015
6	0.22	-1.51	0.299	28	0.43	-0.84	0.015
7	0.25	-1.39	0.175	29	0.50	-0.69	0.075
8	0.28	-1.27	0.093	30	0.50	-0.69	0.075
9	0.28	-1.27	0.093	31	0.50	-0.69	0.075
10	0.28	-1.27	0.093	32	0.50	-0.69	0.075
11	0.28	-1.27	0.093	33	0.50	-0.69	0.075
12	0.28	-1.27	0.093	34	0.50	-0.69	0.075
13	0.28	-1.27	0.093	35	0.50	-0.69	0.075
14	0.30	-1.20	0.056	36	0.54	-0.62	0.123
15	0.31	-1.17	0.041	37	0.54	-0.62	0.123
16	0.31	-1.17	0.041	38	0.64	-0.45	0.272
17	0.31	-1.17	0.041	39	0.65	-0.43	0.288
18	0.32	-1.14	0.030	40	0.70	-0.36	0.373
19	0.35	-1.05	0.007	41	0.71	-0.34	0.391
20	0.36	-1.02	0.003	42	0.74	-0.30	0.444
21	0.40	-0.92	0.003	43	0.80	-0.22	0.554
22	0.40	-0.92	0.003		Σ =	-41.60	6.399

Table 4-3. Example 1 Lilliefors goodness-of-fit test

i	r_i	S	$F_{dm}(r_i)$	$\|F\text{-}S\|$	i	r_i	S	$F_{dm}(r_i)$	$\|F\text{-}S\|$
	0.19	0.000	0.038	0.038	23	0.4	0.535	0.552	0.017
2	0.19	0.047	0.038	0.009		0.42	0.535	0.601	0.066
	0.2	0.047	0.050	0.004	24	0.42	0.558	0.601	0.043
3	0.2	0.070	0.050	0.020		0.43	0.558	0.624	0.066
	0.22	0.070	0.081	0.011	28	0.43	0.651	0.624	0.027
6	0.22	0.140	0.081	0.059		0.5	0.651	0.759	**0.108**
	0.25	0.140	0.142	0.002	35	0.5	0.814	0.759	0.055
7	0.25	0.163	0.142	0.021		0.54	0.814	0.816	0.002
	0.28	0.163	0.217	0.054	37	0.54	0.860	0.816	0.045
13	0.28	0.302	0.217	0.085		0.64	0.860	0.909	0.049
	0.3	0.302	0.272	0.030	38	0.64	0.884	0.909	0.025
14	0.3	0.326	0.272	0.053		0.65	0.884	0.915	0.032
	0.31	0.326	0.301	0.025	39	0.65	0.907	0.915	0.008
17	0.31	0.395	0.301	0.094		0.7	0.907	0.941	0.034
	0.32	0.395	0.330	0.066	40	0.7	0.930	0.941	0.011
18	0.32	0.419	0.330	0.089		0.71	0.930	0.945	0.015
	0.35	0.419	0.416	0.002	41	0.71	0.953	0.945	0.008
19	0.35	0.442	0.416	0.025		0.74	0.953	0.956	0.003
	0.36	0.442	0.445	0.003	42	0.74	0.977	0.956	0.021
20	0.36	0.465	0.445	0.020		0.8	0.977	0.972	0.005
	0.4	0.465	0.552	0.087	43	0.8	1.000	0.972	0.028
								Max =	**0.11**

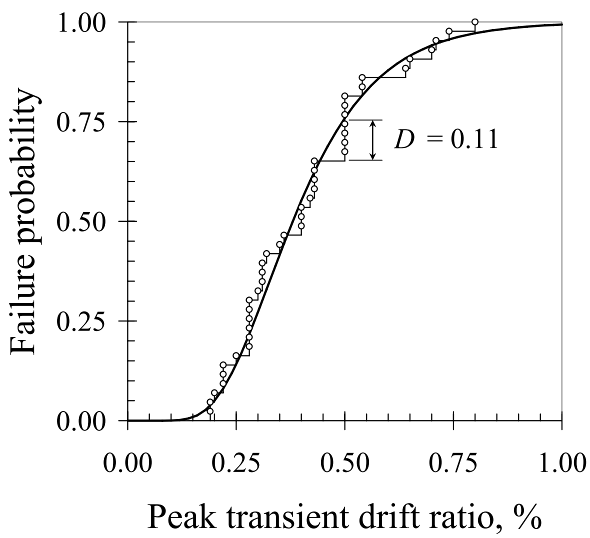

Figure 4-1. Example 1 fragility function with data and sample cumulative distribution function

Example 2. Method B. Consider the (imaginary) damage statistics shown in Figure 4-2. The figure depicts performance of motor control centers (MCCs) observed after various earthquakes in 45 facilities. Each box represents one specimen. Crosshatched boxes represent MCCs that experienced noticeable earthquake effect such as shifting but that remained operable. Black boxes represent those that were found to be inoperable following the earthquake. Each stack of boxes represents one facility. Calculate the fragility function using *PGA* as the *EDP*, binning between halfway points between *PGA* values shown in the figure.

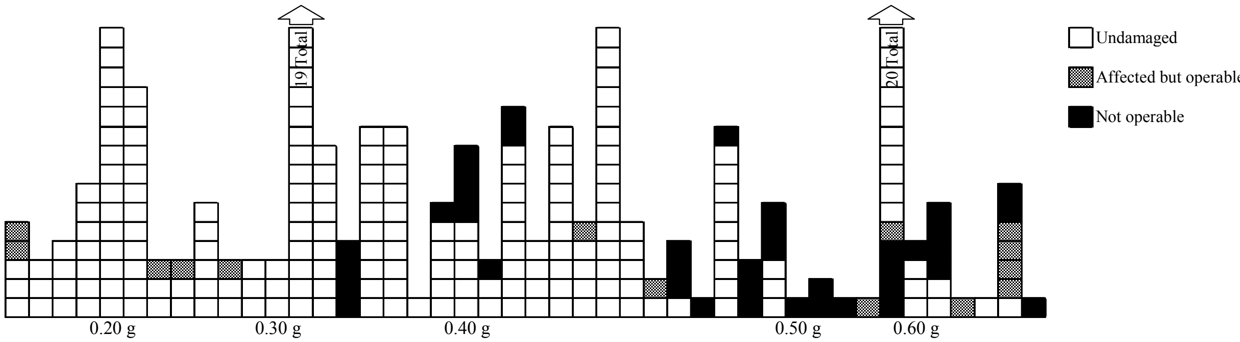

Figure 4-2. Example 2 damage data

Solution. The number of bins, N, and the lower *EDP* bounds a_j, are dictated by the available data: $N = 5$ bins with lower bounds of 0.15g, 0.25g, etc. The values of M_j and m_j are found by counting all boxes and black boxes, respectively, in Figure 4-2, in each bin, and are shown in Table 4-4. The value of M is found by summing: $M = \Sigma M_j = 260$. Values x_j and y_j are calculated as $x_j = \ln(\bar{r}_j)$, and $y_j = \Phi^{-1}((m_j+1)/(M_j+1))$. Average values are calculated as shown: $\bar{x} = -0.99$, $\bar{y} = -1.05$, according to Equation 18. For each bin, the values of $x_j - \bar{x}$ and $y_j - \bar{y}$ are calculated as shown.

Table 4-4. Example 2 solution data

j	a_j (g)	$\bar{r}_j$ (g)	M_j	m_j	x_j	y_j	$x_j - \bar{x}$	$y_j - \bar{y}$	$(x_j - \bar{x})^2$	$(x_j - \bar{x})(y_j - \bar{y})$
1	0.15	0.2	52	0	-1.61	-2.08	-0.623	-1.031	0.388	0.642
2	0.25	0.3	48	4	-1.20	-1.27	-0.217	-0.223	0.047	0.049
3	0.35	0.4	84	8	-0.92	-1.25	0.070	-0.202	0.005	-0.014
4	0.45	0.5	35	15	-0.69	-0.14	0.294	0.907	0.086	0.266
5	0.55	0.6	41	12	-0.51	-0.50	0.476	0.549	0.226	0.261
Σ =			260		-4.93	-5.23			0.753	1.204
Average =					-0.99	-1.05				

Then, β and x_m are calculated as shown in Equation 17:

$$
\begin{aligned}
\beta &= \frac{\sum_{j=1}^{N}\left(x_j - \bar{x}\right)^2}{\sum_{j=1}^{N}\left(x_j - \bar{x}\right)\left(y_j - \bar{y}\right)} \\
&= \frac{0.753}{1.204} \\
&= 0.63 \\
x_m &= \exp\left(\bar{x} - \bar{y}\beta\right) \\
&= \exp\left(-0.99 + 1.05 \cdot 0.63\right) \\
&= 0.72g
\end{aligned}
$$

The results can be checked by plotting y versus x and fitting a line, as shown Figure 4-3: β is the inverse of the slope of the trendline, 1/1.60 = 0.62, and x_m is the value of r at which the line has a y-value of 0, i.e., $x_m = \exp(-0.53/1.60) = 0.72$.

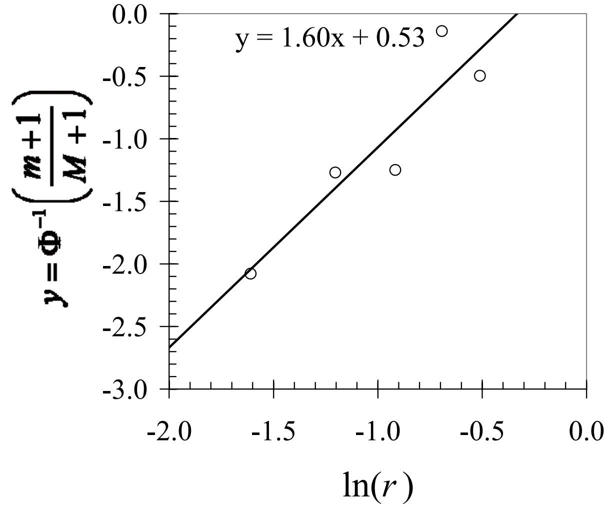

Figure 4-3. Checking Example 2 results

Example 3. Alternative Method B. Fit a lognormal cumulative distribution function to on the data from Example 2 using the alternative method presented in C2.2.

Solution. Here,

M	= number of specimens observed
	= 260
i	= index of specimens, $i \in \{1, 2, \ldots M\}$
r_i	= PGA to which specimen i was subjected, shown in Figure 4-2, $r_i \in \{0.2g, 0.3g, \ldots 0.6g\}$
f_i	= failure indicator for specimen i, shown in Figure 4-2
	= 1 if specimen i failed (not operable)
	= 0 otherwise
Φ	= standard normal (Gaussian) cumulative distribution function

Per Equation 31, the objective is to find x_m and β to minimize the average squared error ε such that:

$$\varepsilon = \frac{1}{M}\sum_{i=1}^{M}\left(f_i - \Phi\left(\frac{\ln\left(r_i/x_m\right)}{\beta}\right)\right)^2$$

$$x_m > 0$$

$$\beta > 0$$

For example, specimen 101 failed by the time it had experienced r = 0.30g. Denoting by ε_i the contribution to the error term ε from specimen i,

$$\varepsilon_{101} = \left(f_{101} - \Phi\left(\frac{\ln\left(r_{101}/x_m\right)}{\beta} \right) \right)^2$$
$$= \left(1 - \Phi\left(\frac{\ln\left(0.30/x_m\right)}{\beta} \right) \right)^2$$

Summing the ε_i for i = 1, 2, … 260, finding x_m and β that minimize ε, and omitting the data for brevity, it is found that

x_m = 0.74g
β = 0.59

These values compare well with Example 3, which produced x_m = 0.72 and β = 0.62.

Example 4. Second Alternative Method B. Consider the following data on hydraulic elevator failures in the Loma Prieta and Northridge Earthquakes. Using *PGA* as the *EDP* and taking each facility as a separate bin, create a fragility function for hydraulic elevators using the second alternative method provided in Section C2.2.

Table 4-5. Summary of hydraulic elevator failure data

Location	***PGA* (g)**	**No. exposed**	**No. damaged**	**Earthquake**
Stanford University	0.26	77	4	Loma Prieta
Valley Presbyterian	0.38	4	2	Northridge
St Johns Hospital Main Wing	0.50	1	1	Northridge
St Johns Hospital South Wing	0.50	1	0	Northridge
St Johns Hospital Mental Health Center	0.50	2	2	Northridge
Cedars Sinai Becker	0.26	1	0	Northridge
Cedars Sinai Cancer	0.26	2	0	Northridge
Northridge Medical Center	0.45	2	1	Northridge
USC Medical Center	0.25	1	0	Northridge
Total		91	10	

Solution. Here, each location is treated as a bin; x_j = *PGA* at location j, M_j = number of elevators at location j, m_j = number of damaged elevators at location j, $y_j = m_j/M_j$, ε_j^2 is calculated as

$$\varepsilon_j^2 = \frac{M_j}{M}\left(y_j - \Phi\left(\frac{\ln\left(x_j/x_m\right)}{\beta_r} \right) \right)^2$$

and

$$\varepsilon^2 = \sum_{j=1}^{9} \varepsilon_j^2$$

j Location	x_j (g)	M_j	m_j	M_j/M	y_j	Φ	ε^2_j
1. Stanford University	0.26	77	4	0.85	0.05	0.06	9.46E-06
2. Valley Presbyterian	0.38	4	2	0.04	0.50	0.40	4.71E-04
3. St Johns Hospital Main Wing	0.50	1	1	0.01	1.00	0.76	6.41E-04
4. St Johns Hospital South Wing	0.50	1	0	0.01	0.00	0.76	6.32E-03
5. St Johns Hospital Mental Health Center	0.50	2	2	0.02	1.00	0.76	1.28E-03
6. Cedars Sinai Becker	0.26	1	0	0.01	0.00	0.06	3.36E-05
7. Cedars Sinai Cancer	0.26	2	0	0.02	0.00	0.06	6.72E-05
8. Northridge Medical Center	0.45	2	1	0.02	0.50	0.63	3.70E-04
9. USC Medical Center	0.25	1	0	0.01	0.00	0.04	1.89E-05
	***M* =**	91				**Total**	9.22E-03

The values of x_m and β_r that minimize ε^2 are x_m = 0.41g and β_r = 0.28. It can be assumed that not all specimens were in the same configuration, had the same installation conditions, or experienced the same loading history, so β_u = 0 and

x_m = 0.41g
β = 0.28

Example 5. Method C. Ref [20] performed full-scale shake-table tests on ceiling systems with a variety of lateral restraint systems. Ten tests simulated conditions with a perimeter wall that provided a boundary, and with the ceiling attached to the perimeter. Peak ceiling acceleration (*PCA*) and peak diaphragm acceleration (*PDA*) from 9 of these tests are recorded in Table 4-6. Calculate the fragility function in terms of (a) *PDA*, and (b) *PCA*.

Table 4-6. Example 5 ceiling test data

ID	Test, run	*PDA* (g)	PCA (g)	Failure
5	7-2	0.39	1.17	FALSE
7	6-1	0.48	1.82	FALSE
8	4-1	0.49	0.82	FALSE
10	5-1	0.51	1.08	FALSE
11	7-3	0.52	1.48	FALSE
12	6-3	0.69	1.81	FALSE
13	7-4	0.76	1.70	INCIPIENT
14	5-5	0.79	2.56	FALSE
16	6-4	1.03	2.43	INCIPIENT

Solution. (a) Here, r_i = PDA for specimen i, r_{max} = 1.03g, r_d = 0.76g, 0.7r_{max} = 0.72g, r_a = min(0.76g, 0.72g) = 0.72g, M_A = 1, M_B = 0, and M_C = 2. By Equation 19,

$$
\begin{aligned}
S &= (0.5M_C + 0.1M_B)/(M_A + M_B + M_C) \\
&= (0.5\cdot 2 + 0.1\cdot 0)/(1 + 0 + 2) \\
&= 0.33
\end{aligned}
$$

Since $M_B + M_C > 0$, $r_m = 0.5\cdot(r_{max} + r_a)$ = 0.88g. From Table 2-1, $S > 0.3$, so $F_{dm}(r_m)$ = 0.4. From Equation 20, β = 0.4 and $x_m = 1.11r_m$ = 0.97g peak diaphragm acceleration.

(b) Here, r_i = peak ceiling acceleration for specimen i, r_{max} = 2.56g, r_d = 1.70g, 0.7r_{max} = 1.79g, r_a = min(1.70g, 1.79g) = 1.70g, M_A = 3, M_B = 0, and M_C = 2. By Equation 19,

$S = (0.5M_C + 0.1M_B)/(M_A + M_B + M_C)$
$= (0.5\cdot 2 + 0.1\cdot 0)/(3 + 0 + 2)$
$= 0.2$

From Table 2-1, $0.15 < S \le 0.3$, so $F_{dm}(r_m) = 0.2$. Since $M_B + M_C > 0$, $r_m = 0.5\cdot(r_{max} + r_a) = 2.13$g. From Equation 20, $\beta = 0.4$ and $x_m = 1.40\cdot r_m = 3.0$g peak ceiling acceleration.

Example 6. Method D. Overturning of tall slender objects such as bookcases can disrupt normal building operations and in some cases endanger life safety. Ref [21] proposes that if the coefficient of static friction between the floor and an object (of width B and height H), is in excess of B/H, then in earthquake shaking, rocking will occur (as opposed to sliding), and the object can overturn when the floor acceleration *and* velocity exceed certain threshold values:

$a_0 = B/H$ (units of gravity)
$v_0 = 10B/H^{0.5}$ (units of cm and sec)

Create two fragility function for bookcases with B = 12 in, H = 72 in, for a building with T = 0.3 sec. The first fragility function, for damage state dm = 1, is for withdrawal of the screw from the wall. The nominal withdrawal capacity of the screw in tension is 80 lb. Also calculate the fragility function for dm = 2, overturning of the bookcase. What if the building has T = 1.0 sec? Assume the 50-lb bookcase is loaded with 200lb of books, that the coefficient of static friction between books and bookcase is 0.5, and that the bookcase is secured to gypsum wallboard partition by a fabric strap at the top secured to the wall by a no-8 self-drilling screw fixed into the 25-ga. metal stud behind through 5/8-in wallboard. What is the fragility function for dm = 1 if the screw misses the stud (withdrawal capacity 15 lb)?

Solution. First consider dm = 1. The reactive mass is 150lb acting at a height of 36 in, opposed by a screw with pullout capacity of 80 lb acting at a height of 72 in. Pullout occurs when

$$a \cdot 150lbm \cdot 36in > 80lbf \cdot 72in$$
$$a > 1.1g$$

Then by Equation 21,

$x_m = 0.92\cdot 1.1$g = 1.0g peak diaphragm acceleration
$\beta = 0.4$

If the screw misses the stud, pullout occurs when $a > 15\cdot 72/(150\cdot 36)$, or 0.2g, and

$x_m = 0.92\cdot 0.2$g = 0.18g peak diaphragm acceleration
$\beta = 0.4$

Next consider overturning of the unanchored bookcase (dm =2; the bookcase must have passed through dm = 1 already). Here, B/H = 0.167. For common interfaces between a bookcase and a floor finish (e.g., steel or wood or carpet), $\mu >> 0.167$, so it can be assumed that bookcases will rock rather than slide. Assuming sinusoidal excitation of amplitude a at angular frequency ω, overturning occurs when

$$\left|a\sin(\omega t)\right| > a_0 \ and \ \left|a\cos(\omega t)/\omega\right| > v_0$$

Making units explicit and expressing v_0 in terms of a_0,

$$
\begin{aligned}
a_0 &= 1g \cdot \frac{B}{H} \\
v_0 &= 10\frac{cm^{0.5}}{\sec}\frac{B}{H^{0.5}} \\
&= 10\frac{cm^{0.5}}{\sec} \cdot \frac{1g}{981\frac{cm}{\sec^2}} \cdot \frac{B}{H^{0.5}} \\
&= 0.0102\frac{g \cdot \sec}{cm^{0.5}}\frac{B}{H^{0.5}} \\
&= 0.0102 a_0 H^{0.5}\frac{\sec}{cm^{0.5}}
\end{aligned}
$$

overturning occurs when

$$
\begin{aligned}
&\left|a\sin(\omega t)\right| > a_0 \ \ and \ \left|a\cos(\omega t)/\omega\right| > 0.010 a_0 H^{0.5} \\
&a^2\sin^2(\omega t) > a_0^2 \ \ and \ a^2\cos^2(\omega t) > \left(0.010 a_0 \omega H^{0.5}\right)^2 \\
&a^2\left(\sin^2(\omega t)+\cos^2(\omega t)\right) > a_0^2\left(1+0.0102\omega H^{0.5}\right)^2 \\
&a^2 > a_0^2\left(1+0.0102\omega H^{0.5}\right)^2 \\
&a > \frac{B}{H}\left(1+0.0102\omega H^{0.5}\right) \\
&a > \frac{1}{6}\cdot\left(1+0.0102\cdot\frac{2\pi}{T}\cdot 183^{0.5}\right) \\
&a > \left(0.167+\frac{0.144}{T}\right)
\end{aligned}
$$

So for a building with T = 0.3 sec, overturning of the unanchored bookcase occurs when $PDA \geq 0.65$g. For the case where the screw is anchored in the stud, dm = 2 will be exceeded when dm = 1 is exceeded, and thus for dm = 2,

x_m = 1.0g
β = 0.4

For T = 0.3 sec and the screw misses the stud, the overturning capacity governs dm = 2, and

x_m = 0.92·0.65g = 0.60g
β = 0.4

For the building with 1-sec period, overturning of the unanchored bookcase occurs when $PDA \geq 0.31$g. If the screw is anchored in the stud, overturning will occur only after screw pullout, and again for dm = 2,

x_m = 1.0g
β = 0.4

For the case of T = 1.0 sec and the screw misses the stud, the overturning capacity governs dm = 2, and

$x_m = 0.92 \cdot 0.31\text{g} = 0.29\text{g}$
$\beta = 0.4$

Example 7. Method E. Stone cladding on the exterior of office buildings is potentially subject to falling in earthquakes. Consider 2-ft x 2-ft x 1-3/16-in granite veneer adhered to a concrete masonry unit substrate with thin-bed mortar (liquid latex mixed with Portland cement, 100% coverage). Create a fragility function for the probability that any given panel would fall from the building, as a function of the peak transient drift ratio of the story on which the panel is applied.

Solution. Figure 2-1 was used to solicit judgment from three engineers on the fragility of the component, using the following definitions.

Component name:	Granite cladding 1
Component definition:	2-ft x 2-ft x 1-3/16-in granite veneer adhered to CMU substrate with thin-bed mortar (liquid latex mixed with Portland cement, 100% coverage.)
Damage state name:	Falling
Damage state definition:	*Any given cladding panel* becomes delaminated from CMU substrate and falls
Relevant *EDP*:	*PTD*
Definition of *EDP*:	Peak transient drift ratio of story and column line on which panel is adhered

Responses are shown in columns 2, 3, and 4 of the following:

(1) *Response i*	**(2) Expertise w_i**	**(3) Median x_{mi}**	**(4) Lower bound x_{li}**	**(5) $w_i^{1.5}$**	**(6) $w_i^{1.5} \cdot x_{mi}$**	**(7) $w_i^{1.5} \cdot x_{li}$**
1	2	0.003	0.0015	2.83	0.0085	0.0042
2	1	0.005	0.001	1.00	0.0050	0.0010
3	2	0.010	0.005	2.83	0.0283	0.0141
			$\sum =$	6.66	0.0418	0.0194

By Equations 23, 24, and 25,

$$x_m = \frac{\sum_{i=1}^{N} w_i^{\alpha} x_{mi}}{\sum_{i=1}^{N} w_i^{\alpha}} = \frac{0.0418}{6.66} = 0.63\% \qquad x_l = \frac{\sum_{i=1}^{N} w_i^{\alpha} x_{li}}{\sum_{i=1}^{N} w_i^{\alpha}} = \frac{0.0194}{6.66} = 0.29\% \qquad \beta = \frac{\ln\left(\frac{x_m}{x_l}\right)}{1.28} = \frac{\ln\left(\frac{0.0063}{0.0029}\right)}{1.28} = 0.60$$

Example 8. Method U. Consider Example 4, Method C. One additional test is reported in Ref [20]: a ceiling collapse in a test of a ceiling with perimeter attachment, where the collapse occurred at or below *PDA* = 1.99g. Update the fragility function from Example 4a.

Solution. From Example 5a,

$x_m =$ 1.14
$\beta =$ 0.40

The new data point is:

i	r_i	f_i
1	1.99	1

From Equation 30,

	j				
	1	**2**	**3**	**4**	**5**
x_{mj}	1.14	0.70	1.86	1.14	1.14
β_j	0.40	0.40	0.40	0.26	0.54
w_j	0.333	0.167	0.167	0.167	0.167

For example,

$x_{m2} = 1.14\exp(-1.22 \cdot 0.40) = 0.70$
$\beta_4 = 0.64 \cdot 0.40 = 0.26$

From Equation 29,

$L(i,j)$	j				
i	**1**	**2**	**3**	**4**	**5**
1	0.92	1.00	0.57	0.99	0.85

For example,

$L(1,1) = \Phi(\ln(1.99/1.14)/0.4) = 0.92$

Calculating the terms needed for Equation 28, and noting that since $M = 1$, $\Pi_i L(i,j) = L(1,j)$

	j				
	1	**2**	**3**	**4**	**5**
$w_j \Pi_i L(i,j)$	0.31	0.17	0.09	0.16	0.14

For example,

$w_1 L(1,1) = 0.333 \cdot 0.92 = 0.31$

Evaluating the denominator for Equation 28,

$\Sigma_j w_j \, \Pi_i L(i,j) =$ 0.87

Evaluating Equation 28 for each j yields the new weights w'_j and the terms needed for Equation 27:

	j				
	1	**2**	**3**	**4**	**5**
W'_j	0.35	0.19	0.11	0.19	0.16
$w'_j \ln(x_{mj})$	0.05	-0.07	0.07	0.02	0.02
$w'_j \beta_j$	0.14	0.08	0.04	0.05	0.09

For example, $w'_1 = 0.31/0.87 = 0.35$; $w'_1\ln(x_{m1}) = 0.35\ln(1.14) = 0.05$; and $w'_1\beta_1 = 0.35 \cdot 0.40 = 0.14$. Summing the second row,

$$\Sigma_j w'_j \ln(x_{mj}) = \quad 0.09$$

Finally, evaluating Equation 27 yields the updated parameters of the fragility function:

$$\mathbf{x'}_m = \quad 1.10$$
$$\boldsymbol{\beta'} = \quad 0.40$$

SECTION 4 COMMENTARY

In Example 1, note well that the sample CDF is stepped, with a zero slope between different levels of *EDP*. The value of D is therefore not necessarily the maximum vertical distance between the smooth fragility function and a data point, but may be larger.

Examples 2 and 3 employ Ref [22] for data on M_i and r_i, and for the depiction of specimen damage. However, the damage data, m_i, are simulated.

In Example 4, failure data come from [23] and [24]. Failure is defined as experiencing any of the following: damage to controls, the elevator entrance, the car door, car guide shoes, cab stabilizers, cab interior, equipment anchorage, hydraulic cylinder or piping, or snagged ropes and traveling cables. Facilities were geolocated using [25]. *PGA* estimates are interpolated from the two nearest strong-motion instruments, using [26] and [27].

In Example 5, Ref [20] reported that splay-wire bracing and compression struts appear never to be activated. Only perimeter capture and attachment (where present) appear to be relevant to ceiling failure.

In Example 6, the withdrawal capacity of screws from metal stud is taken from [28], and assumes a factor of safety of 1.33. In the example, anchorage failure is treated as a distinct damage state, as anchorage failure involves economic cost (repair of the partition), is not necessarily synonymous with overturning, and overturning has different consequences (threatening life safety).

In Example 8, the updated parameters of the fragility function change little because there is little information in the failure of a specimen failed subjected to $1.75x_m$. Had the specimen *not* failed at that level of excitation, the parameters would have changed more: they would have been $x'_m = 1.5$g and $\beta = 0.43$.

5 OTHER ISSUES

This section addresses issues not dealt with earlier, particularly the correlation of damage between different components in a building.

5.1 CORRELATION OF DAMAGE

It may be that capacity of a particular component type varies between construction contractors, between crews working for that contractor, and perhaps days of the week for a given crew. However, until test or other empirical data are produced to create and defend a model of damage correlation as a function of these or other parameters, it will be assumed here that damage between different components is independent, conditioned on *EDP*.

Rationale. In dealing with nuclear power plant risk studies, the authors of [29] argue that nonzero correlation between damage to different components may exist, even conditioned on *EDP.* They recommend assuming perfect correlation if the components are of the same type (here, the same taxonomic group), same manufacturer (here, same construction contractor), and located at the same level and with the same orientation (here, the same *EDP*). Zero correlation should be assumed if none of these conditions exist. If some but not all conditions exist, they recommend using either perfect correlation or zero correlation, whichever is more conservative. However, in the present application, conservatism is not an objective. Furthermore, data for the fragility functions used here are often developed from specimens constructed by a single crew of a single construction contractor in a laboratory and subjected to the same excitation (same *EDP*), which would argue for perfect correlation. Under perfect damage correlation, β = 0, yet these tests they tend to yield $0.2 \leq \beta \leq 0.6$. Finally, the present application of PBEE aims at simplicity. In summary, it is recommended that correlation be ignored because of (1) no desire for conservative loss estimate, (2) test data that suggest substantially less than perfect correlation, and (3) preference for simplicity.

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