TECHNICAL NOTE

Cracking an Open Safe: Uncertainty in HAZUS-Based Seismic Vulnerability Functions

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The "cracking an open safe" methodology has been used to tabulate HAZUS-based seismic vulnerability as functions of structure-independent intensity, while avoiding iteration in the structural analysis. The vulnerability functions give mean damage factor (MDF, defined here as mean repair cost as a fraction of replacement cost) versus 5%-damped elastic spectral acceleration response at 0.3-second and 1.0-second periods, for every combination of occupancy type, model building type, design level, magnitude, distance, site soil classification, etc. Like HAZUS-MH, these prior seismic vulnerability functions give no estimate of uncertainty in damage factor. The coefficient of variation (COV) of damage factor is readily calculated by taking advantage of the fact that that at any level of excitation there is a probability mass function of damage state and an implicit distribution of repair cost conditioned on damage state. COV is calculated here for each combination of occupancy type, model building type, etc., tabulated alongside MDF, and the tables presented for public use at www.risk-agora.org. It is found that a HAZUS-based COV generally decreases with increasing MDF (as has been observed using other analytical vulnerability methods), and the standard deviation of damage factor generally increases with increasing MDF. [DOI: 10.1193/1.3459153]

INTRODUCTION

In prior work (Porter 2009a, b), the open safe of the HAZUS-MH seismic vulnerability methodology was "cracked" to calculate tables of mean damage factor versus structure-independent 5%-damped spectral acceleration response at 0.3-sec and 1.0-sec periods. The open-safe metaphor refers to the fact that the HAZUS-MH methodology has been thoroughly documented and all its parameter values made available for public use; see especially Kircher and Whitman (1997) and NIBS and FEMA (2003). However, the HAZUS-MH developers do not offer the resulting seismic vulnerability functions in a tabular or graphical form plotted against a structure-independent intensity measure, which can be very inconvenient if one wishes to perform societal risk estimation outside of the HAZUS-MH software. Hence the need to crack the safe. Furthermore, the HAZUS-MH methodology typically involves iteration to estimate structural response by the capacity spectrum method. The cracking-an-open-safe methodology offers a technique to avoid iteration while still honoring the underlying methodology for structural analysis.

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Here, "mean damage factor" refers to the expected value of repair cost as a fraction of replacement cost. In calculating this expected value, the HAZUS-MH methodology propagates, in an approximate way, three sources of uncertainty: ground-motion intensity (not in the sense of Modified Mercalli Intensity, but a vector-valued structure-independent spectral acceleration response) given magnitude and distance, structural response given intensity, and damage given structural response.

Having the mean damage factor available for a particular building in a particular location is clearly very useful and often sufficient, especially when one is dealing with an expected-value problem such as the benefit-cost ratio for a risk-mitigation policy. But what if one wants to know something about the loss exceedance curve (frequency with which various levels of loss are exceeded for a given property), which requires accounting for the uncertainty in loss conditioned on intensity? Estimating the mean damage factor is not enough. The problem addressed here is to estimate the coefficient of variation (denoted here by COV) of damage factor at the same intensity values as the mean damage factor. COV refers to the standard deviation divided by the mean value.

RECAP METHODOLOGY FOR EVALUATING MEAN DAMAGE FACTOR

Before addressing the question of calculating COV versus intensity it is useful to recap the cracking-an-open-safe method. The reader is referred to Porter (2009a, b) for details. Both the HAZUS-MH and cracking-an-open-safe methodologies use the capacity spectrum method to estimate structural response (CSM, Freeman 1998). This pseudostatic nonlinear procedure idealizes a building as a single-degree-of-freedom nonlinear damped harmonic oscillator with a three-part pushover curve: linear up to a yield point (D_y, A_y), perfectly plastic past an ultimate point (D_u, A_u), and a portion of an ellipse between the two. The effective damping ratio is estimated as the sum of an elastic damping ratio and a fraction of the hysteretic damping, where the fraction, κ, is a function of earthquake magnitude (and indirectly, duration) to reflect pinching of the idealized hysteresis loop. Structural response is parameterized as the spectral displacement (S_d, in inches) and spectral acceleration (S_a, in g's) of the idealized oscillator. It is estimated as the point—called the performance point—where the pushover curve intersects the idealized demand spectrum with the same effective damping ratio B_{eff}.

The idealized demand spectrum has two parts: a constant-acceleration portion and a constant-velocity portion, both adjusted to account for site soil amplification (F_a in the constant-acceleration region and F_v in the constant-velocity region) and effective damping (using R_A in the constant-acceleration region and R_V in the constant-velocity region). Figure 1 summarizes the methodology. Equations 1–7 give the equations for the demand spectrum. In Equation 6, *Area* is the area of one idealized full hysteresis loop whose upper right hand corner is the performance point, and B_E is the elastic damping ratio. See Porter (2009a) for the pushover curve, especially for the ellipse portion between yield and ultimate.

$$S_a = S_S F_a / R_A \quad 0 < T \leq T_{AVD} \tag{1}$$

$$S_a = S_1 F_v / (R_V T) \quad T_{AVD} \leq T \tag{2}$$

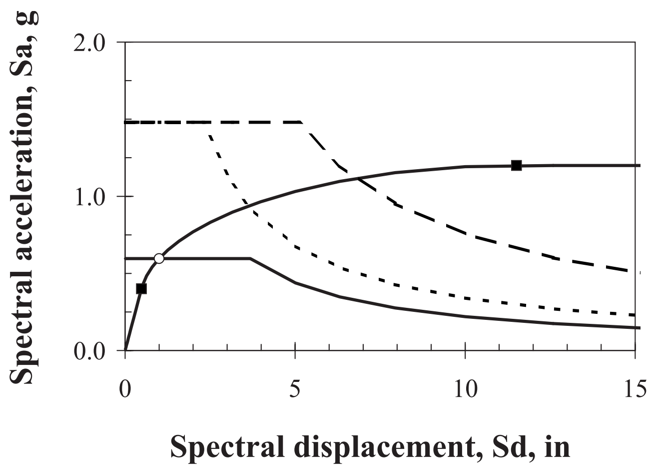

Figure 1. Capacity spectrum method of structural analysis as intended for HAZUS.

$$T = 0.32\sqrt{S_d/S_a} \tag{3}$$

$$R_A = 2.12/(3.21 - 0.68\ \ln[100B_{eff}]) \tag{4}$$

$$R_V = 1.65/(2.31 - 0.41\ \ln[100B_{eff}]) \tag{5}$$

$$B_{eff} = B_E + \kappa(Area/(2\pi S_d S_a)) \tag{6}$$

$$Area \approx 4S_a(S_d - S_a/(A_y/D_y)) \tag{7}$$

Damage is estimated by inputting structural response (S_d, S_a) into lognormal fragility functions for three generalized buildings components (structural, nonstructural drift-sensitive, and nonstructural acceleration-sensitive) and each of four qualitative damage states (slight, moderate, extensive and complete), as shown in Equation 8 for the uncertain structural damage state D_s. In the equation, Φ is the cumulative standard normal distribution function, θ_i and β_i are parameters of the distribution for damage state i, and d is a particular value of D_s. Nonstructural drift-sensitive damage D_{nd} and nonstructural acceleration-sensitive damage D_{na} are estimated similarly. Repair cost is then estimated as the sum of component damage state probabilities times the mean repair cost given component damage, as shown in Equation 9. In the equation, $L_{s,d}$, $L_{nd,d}$, and $L_{na,d}$ represent the mean damage factor (repair cost as a fraction of total replacement cost V) for the structural, nonstructural drift-sensitive and nonstructural acceleration-sensitive components, respectively, given that they are in damage state d.

$$
\begin{aligned}
P[D_s = d | S_d = x] &= \Phi\left(\frac{\ln[x/\theta_d]}{\beta_d}\right) - \Phi\left(\frac{\ln[x/\theta_{d+1}]}{\beta_{d+1}}\right) \quad 1 \leqslant d \leqslant 3 \\
&= \Phi\left(\frac{\ln[x/\theta_4]}{\beta_4}\right) \qquad\qquad\qquad\qquad d = 4
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
E[L] = V\Bigg(& \sum_{d=1}^{4} P[D_s = d | S_d = x] L_{s,d} + \sum_{d=1}^{4} P[D_{nd} = d | S_d = x] L_{nd,d} \\
& + \sum_{d=1}^{4} P[D_{na} = d | S_a = y] L_{na,d} \Bigg)
\end{aligned} \tag{9}
$$

A main difference between the HAZUS-MH approach and the cracking-an-open-safe approach is that the latter begins by selecting an S_d value for the performance point. One can then work backwards to calculate S_a, T, B_{eff}, R_A, R_V, F_a, F_v, S_S and S_1, which requires no iteration, and forwards to $E[L]$. By repeating the process at many values of S_d, one can create a lookup table relating $S_S F_a$ and $S_1 F_v$, which are the 5%-damped values of spectral acceleration response accounting for site soil, to $E[L]$, for a given combination of magnitude, distance, etc. The HAZUS-MH approach, by contrast, starts with S_S and S_1, calculating forward to the performance point (S_d, S_a), which *does* tend to require iteration, and thence to $E[L]$. The computational demands of the iteration can be significant for a large portfolio or a probabilistic risk assessment. Having a vulnerability function lookup table greatly reduces the computational demands. Informal experiments by the USGS's PAGER project team (PAGER stands for Prompt Assessment of Global Earthquakes for Response; Porter et al. 2008) show that a scenario loss estimate for a large portfolio—thousands or tens of thousands of properties—can take seconds rather than hours typically required by HAZUS-MH.

METHOD FOR CALCULATING COV

With this background in mind, the calculation of COV is straightforward. At the end of Equation 8, we have the probabilistic damage state of the building, accounting for the uncertainties mentioned earlier. We will need to account for uncertainty in loss given damage state, which is nowhere stated in the HAZUS-MH documentation. NIBS and FEMA (2003) give only the mean loss given damage state, denoted in Equation 9 by $L_{s,d}$, $L_{ns,d}$, and $L_{na,d}$.

It is assumed here that the loss conditioned on damage state is uniformly distributed between lower and upper bounds for each component and damage state. This is an important assumption, but it seems justified: we do not know the true distribution, and can only estimate its bounds. Information theory says that the uniform distribution is the maximum-entropy (minimum-information) distribution under those conditions. That is, to assume any other distribution implies more knowledge—less uncertainty—than seems available, and would produce a lower COV. Now, proceeding under this assumption, the second moment of loss given the performance point (S_d, S_a) is given by Equation 10:

Table 1. Approximate values of a and b in Equation 10, as inferred from NIBS and FEMA (2003) Tables 15.2, 15.3, and 15.4

	Structural		Nonstructural drift-sensitive		Nonstructural acceleration-sensitive	
d	a–b	s	a–b	s	a–b	s
Slight	0–0.01	0.0029	0–0.01	0.0029	0–0.02	0.0058
Moderate	0.01–0.05	0.0115	0.01–0.07	0.0173	0.02–0.10	0.0231
Extensive	0.05–0.15	0.0289	0.07–0.15	0.0231	0.10–0.30	0.0577
Complete	0.15–0.25	0.0289	0.15–0.65	0.1443	0.30–0.50	0.0577

$$
\begin{aligned}
E[L^2] = \sum_{d=1}^{4} P[D_s = d|S_d = x] \cdot \left(\frac{(b_{s,d} - a_{s,d})^2}{12} + L_{s,d}^2 \right) \\
+ \sum_{d=1}^{4} P[D_{nd} = d|S_d = x] \cdot \left(\frac{(b_{nd,d} - a_{nd,d})^2}{12} + L_{nd,d}^2 \right) \\
+ \sum_{d=1}^{4} P[D_{na} = d|S_a = y] \cdot \left(\frac{(b_{na,d} - a_{na,d})^2}{12} + L_{na,d}^2 \right)
\end{aligned} \tag{10}
$$

In Equation 10, $b_{s,d}$ and $a_{s,d}$ refer to the upper and lower bounds of damage factor for the structural component given that it is in damage state d. Similarly the terms $b_{nd,d}$, $a_{nd,d}$ refer to the same bounds for the nonstructural drift-sensitive component, and $b_{na,d}$, $a_{na,d}$ refer to the bounds for the nonstructural acceleration-sensitive component. In each sum, the first term gives the probability of the component being in a given damage state, the second gives the second moment of loss about its central value, and the third term gives the second moment of the central value about zero.

Finally, one can calculate the coefficient of variation of loss by recalling that variance is given by Equation 11 and COV by Equation 12:

$$
Var[L] = E[L^2] - E^2[L] \tag{11}
$$

$$
COV[L] = \frac{\sqrt{Var[L]}}{E[L]} = \frac{\sqrt{E[L^2] - E^2[L]}}{E[L]} = \sqrt{\frac{E[L^2]}{E^2[L]} - 1} \tag{12}
$$

Now the only missing pieces of information are the a and b values of Equation 10. First consider the values of $L_{s,d}$, $L_{nd,d}$, and $L_{na,d}$ proposed in NIBS and FEMA (2003). In general, Tables 15.2, 15.3 and 15.4 of the HAZUS-MH earthquake technical manual assume that repair of slight structural damage costs approximately 0.5% of building replacement cost, moderate generally costs 3%, extensive is approximately 10%, and complete is approximately 20%. For nonstructural drift-sensitive, slight, moderate, extensive and complete are roughly 0.5%, 4%, 11%, and 40%. For nonstructural acceleration-

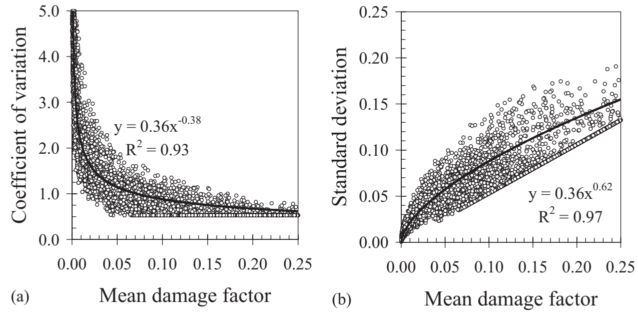

Figure 2. Trends in (a) COV versus MDF and (b) standard deviation versus MDF.

sensitive, the figures are roughly 1%, 6%, 20%, and 40%. It seems reasonable to infer the associated ranges as shown in Table 1. The table also shows the value of the second term in each summand of Equation 10, i.e.,

$$s^2 = \frac{(b-a)^2}{12} \tag{13}$$

Equation 12 was evaluated for every combination of HAZUS-MH's occupancy type, model building type, design level, magnitude (5, 6, 7, or 8), distance (10, 20, 40, or 80 km), seismic domain (plate boundary or continental interior) and NEHRP site soil category (A, B, C, D or E), appended to the seismic vulnerability function tables detailed in Porter (2009b), and posted as a free download at the home page of the Alliance for Global Open Risk Analysis (AGORA 2009; www.risk-agora.org; free registration is required).

The coefficient of variation generally decreases with increasing mean damage factor, as has been observed using other analytical vulnerability methodologies (e.g., Porter et al. 2006), as illustrated in Figure 2. The standard deviation of damage factor (mean times COV) modestly increases with MDF. In the figures, COV has been artificially limited to 0.5 or greater since lower values seem unrealistic, and only samples with nonzero MDF are shown. The values of COV seem generally reasonable, at least on an order-of-magnitude basis, given the fairly broad definitions of the HAZUS-MH model buildings types and the various sources of uncertainty involved. The equations in the figures refer to the trendlines shown overlain on the sample data, and the value R^2 refers to the fraction of the total data variance accounted for by the trendline.

CONCLUSIONS

In previous work, the "open safe" of the HAZUS-MH earthquake vulnerability methodology was cracked to produce tabular seismic vulnerability functions: mean damage factor versus either of two structure-independent intensity measures: 5%-damped spectral acceleration at 0.3-sec or 1.0-sec periods. The seismic vulnerability functions were tabulated for every combination of occupancy type, model building type, design level, seismic domain, magnitude, distance, and NEHRP site soil classification. In the present work it was shown that one can also readily calculate the coefficient of variation of damage factor, useful for probabilistic risk analyses. This appears not to have been done before. The results are available at www.risk-agora.org. Free registration is required for download.

No judgment is offered as to the accuracy of the underlying HAZUS-MH vulnerability parameter values or of its methodology. The capacity spectrum method (CSM) of structural analysis, used in HAZUS-MH, has been extensively critiqued and several alternatives or enhancements have been proposed. See especially Applied Technology Council (2004) or any of several works on second-generation performance-based earthquake engineering that use multiple nonlinear dynamic analyses in loss estimation (e.g., Beck et al. 1999, Porter et al. 2001, or Goulet et al. 2007). However, HAZUS-MH is far more exhaustive than any other public source of seismic vulnerability information in terms of variety of structure types and consideration of strong influences on vulnerability. Until something better comes along, it seems likely that people doing seismic risk analysis at the societal level will find the HAZUS-MH vulnerability information useful.

ACKNOWLEDGMENTS

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