Analytical Derivation of Seismic Vulnerability Functions for Building Classes and Nonstructural Components, Emphasizing Highrise Buildings

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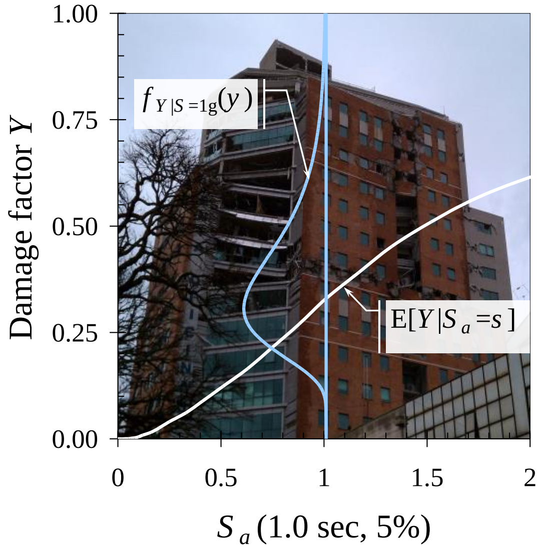

GEM
GLOBAL EARTHQUAKE MODEL

Analytical Derivation of Seismic Vulnerability Functions for Building Classes and Nonstructural Components, Emphasizing Highrise Buildings

Global Vulnerability Consortium

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Version history

Version	Date	Summary
0.9	March 31, 2013	Initial draft
0.91	April 2, 2013	Mention principal limitation of methodology (fragility functions for masonry) in abstract
0.92	April 8, 2013	Complete sample calculations for 1- and 3-index building examples
0.93	October 22, 2013	Make clearer that both collapse and non-collapse are handled per Meslem meeting
0.94	October 28, 2013	Add illustrations of the three choices for uncertainty propagation per Meslem meeting
0.95	October 28, 2013	Add placeholders of structural analysis sections 4.4.3 (new section for pushover) and 4.4.5 (new section for multiperiod scalar IM) TBD by DV; add placeholders for other commentary: D1 approx equivalence of between-buildings and within-building variability TBD by KP; D2 rotation point for fragility functions based on Type-B data, TBD by KP. Add note in section 4.4.4 where KP has to address DV's comments about record selection and CMS; see DV's email today about record selection which has some suggested text.
0.96	December 19, 2013	Added new material on record selection, IM selection, performing dynamic analysis and some optional text (inclusion to be decided by KP) on sampling mechanisms for the "seven index buildings" option.
0.97	April 21, 2014	Respond to Vamvatsikos comments; complete blank appendices

ABSTRACT

A procedure is offered for the analytical derivation of the seismic vulnerability of building classes, that is, probabilistic relationships between shaking and repair cost as a fraction of replacement cost new for a category of buildings. It simulates structural response, damage, and repair cost for the structural and non-structural components that contribute most to construction cost, and then scales up results to account for the components that were not simulated. It does so for a carefully selected sample of building specimens called index buildings whose designs span the domain of up to three features that are believed to most strongly influence seismic vulnerability within the building class. One uses moment matching to combine results for the index buildings to estimate behaviour and variability of the building class. One can simulate non-structural vulnerability alone by ignoring damage and repair cost for structural components.

The procedure has five steps. In Step 1, the analyst defines the asset class with one, three, or seven specimens of the asset class; the specimens are called index buildings. The choice depends on available resources and the rigor with which the analyst wants to address variabilities within the building class and within the performance of an individual index building. Each index building is assigned a particular structural and non-structural design, including number of stories, structural material, lateral load resisting system (LLRS), geometry, and quantities of each of the top 1 or 2 structural component categories and top 5 or 6 non-structural component categories.

Step 2 is to derive story-level vulnerability functions, absent collapse. (The probability of collapse and the cost given collapse is addressed in a later step.) The vulnerability functions express the repair cost of components on the story as a function of story-level excitation (drift, acceleration, or other measures of story-level structural response). Step 3 is to perform a structural analysis at each of many levels of ground motion with the objective of estimating story-level excitation and collapse probability as a function of ground motion. We offer three options for structural analysis, from a very simple approach to multiple nonlinear dynamic structural analyses; the analyst is free to choose among these, again considering available resources and desired rigor.

Step 4 is to derive a building-level vulnerability function by summing story-level losses over stories, factoring up to account for the fact that only the top 6 to 8 structural and non-structural component categories are inventoried, applying the theorem of total probability to consider the probability of collapse. By omitting the top 2 or so structural components, one can create vulnerability functions for only the non-structural components. The vulnerability function is normalized by replacement cost new to depict damage factor as a function of ground motion.

In Step 5, the mean vulnerability function and coefficient of variation of damage factor for the asset class are calculated. The mean damage factor for the asset class is calculated as a weighted average of those of the index buildings. The coefficient of variation is calculated by one of three means: using a proxy from HAZUS in the case of a single index building, as a multiple of the variability of vulnerability between index buildings in the case of three index buildings, or in the case of seven index buildings, by calculating the variance of vulnerability of the weighted sample of index-building-level vulnerability functions, including both between- and within-building variability.

Keywords: analytical; vulnerability; moment matching

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1 Abbreviations and definitions

Most variables are defined near their first use. Other common abbreviations in this document are as follows.

ASCE	American Society of Civil Engineers
ATC	Applied Technology Council
CDF	Cumulative distribution function. Applied to uncertain quantities, gives the probability that the variable will take on a value less than or equal to a particular value, as a function of that particular value.
CUREE	Consortium of Universities for Research in Earthquake Engineering
DF	Damage factor, typically uncertain repair cost divided by replacement cost new.
DP	Demand parameter, a measure of structural response that can be recorded or estimated from the results of a structural analysis. Typical choices are the peak floor acceleration (PFA) and the peak transient interstory drift ratio (PTD), both of which apply to a particular floor (in the case of PFA) or story (in the case of PTD).
E[·]	Expected (mean) value of the quantity in brackets
E.g.	For example
FEMA	Federal Emergency Management Agency
Floor	A floor or roof diaphragm, including the ground floor
Fragility function	A deterministic relationship between the occurrence probability of some undesirable event and a measure of environmental excitation that is causally related to the event. As used here, it generally refers to the probability that a particular damage state occurs or is exceeded in a building component as a function of the value of the demand parameter to which the component is most sensitive. Alternatively, it may refer to the occurrence probability of some whole-building damage state, especially collapse, as a function of ground motion intensity. In either case the fragility function can be interpreted as the cumulative distribution function of the capacity of the component to resist the specified damage state, or the capacity of the building to resist collapse, measure in terms of the demand parameter that related to the damage state.
GEM	Global earthquake model
Geomean	Geometric mean
GMPE	Ground motion prediction equation, a subclass of which is the attenuation relationship, which relates an intensity measure to earthquake source and path parameters such as magnitude, distance, and site conditions, among others.
HAZUS-MH	Hazards United States Multihazards (a software product of the US Federal Emergency Management Agency)
ID	Identifier
I.e.	That is
IM	Intensity measure. As used here, IM refers to a quantity that characterizes environmental excitation to the asset. This work deals solely with earthquake shaking, so IM is limited here to a measure of degree of ground shaking, and more particularly to a scalar instrumental measure.

Index building	A particular member of an asset class, with specified geometry, materials, material properties, structural and nonst ructural components, replacement cost, and other features that would appear on a complete set of construction or as-built documents, include structural, architectural, mechanical, electrical, plumbing drawings and specifications.
IT	Information technology
Joint distribution	The probability density that two or more uncertainties will take on a particular set of values, one for each uncertainty
km	kilometers
Log std deviation	Logarithmic standard deviation, that is, the standard deviation of the natural logarithm of the variable
Loss	A scalar nonnegative measure of repair costs, life-safety impacts, or loss of functionality ("dollars, deaths, or downtime").
MDF	Mean damage factor, the expected value of repair cost as a fraction of replacement cost new.
MDOF	Multi-degree of freedom
MECE	Mutually exclusive and collectively exhaustive
NISTIR	National Institute of Standards and Technology Interagency Report
Param	Parameter
PFA	Peak floor acceleration, generally geometric mean of two orthogonal directions, units of gravity
PMF	Probability mass function
Population	All members of a class, as in all the buildings that belong to a specified asset class.
PTD	Peak transient interstory drift ratio, unitless
RCN	Replacement cost new, that is, the cost to build a replacement for the building on the same site, excluding demolition of an existing building on the site
Ref	Reference, generally meaning a citation for a source of information
Sample	One or more members of a population
SDOF	Single degree of freedom
Story	The space between two floors
Uncertainty	As used here, a variable with an uncertain value, often called a random variable.
US	United States
Vulnerability	As used here, a probabilistic relationship between uncertain degree of loss to a specified component, story, particular building, or asset class as a function of environmental excitation. When applied to repair cost for a building or asset class, loss is typically normalized by replacement cost new, and is referred to as damage factor.

2 Introduction

2.1 Objectives of the methodology

A procedure is offered for the analytical derivation of the seismic vulnerability of buildings. By "seismic vulnerability" is meant a probabilistic estimate of repair cost (as a fraction of replacement cost new) as a function of intensity measure level for an asset class, where "asset class" is generally limited here to categories of buildings, as opposed to bridges, pipelines, sailboats, etc. These guidelines do *not* use "vulnerability" and "fragility" interchangeably. As used here, fragility means the probability of an undesirable outcome occurring given some environmental excitation. These guidelines use the terms "vulnerability function" and "fragility function," but not "vulnerability curve" and "fragility curve." The choice is stylistic, but is the one used here and throughout the products of the GEM Vulnerability Consortium.

The asset class is defined in whatever terms the analyst has selected from among the GEM Taxonomy. For illustration purposes, it is assumed here that the asset class is defined in terms of structural material, lateral load resisting system, height category, occupancy class, and era of construction. These guidelines were developed for highrise buildings, but are believed to be applicable to most kinds of mid- and lowrise buildings as well. These guidelines are flexible: they accommodate simple structural analysis that idealizes the building as a single-degree-of-freedom nonlinear oscillator. They also allow for more-complex methods that can account for higher modes and multiple degrees of freedom.

The principal constraint of the methods presented here is the availability of component fragility functions. As of this writing, the principal gaps in available component fragility functions are those for unreinforced walls constructed of brick, concrete block, clay tile, earth, or stone. Until those fragility functions are created, the analyst interested in deriving building-level fragility functions for unreinforced masonry bearing wall buildings and confined masonry buildings is referred to the companion work by D'Ayala et al. (2014).

Earthquake-induced property losses in buildings are often considered to be composed of three components: structural, non-structural, and contents. The division between these groups can sometimes be ambiguous. (E.g., gypsum wallboard partitions in woodframe buildings can act as structural or nonstructural components; computers in server racks that are permanently fixed to the floor can be seen as nonstructural elements or as contents). This work proposes procedures for developing analytical seismic vulnerability functions for structural and nonstructural components. We will not attempt to draw a crisp distinction between structural, non-structural, and contents, other than a practical one. If a component contributes significantly to the strength and stiffness of the building, it is structural. If it does not contribute significantly to the strength or stiffness of the building, but is typically delivered by the general contractor with new construction or retrofit, it is considered nonstructural. This is important: components that contribute significantly to strength or stiffness but that are not included in the structural analysis are *not* nonstructural; rather their omission from the structural analysis is considered to be an error in modelling on the part of the analyst. Thus, masonry infill walls are structural, even if not considered in the structural design, since they contribute to stiffness. Some component categories may act as structural components in one context and as nonstructural in another, for example gypsum wallboard partitions on metal studs may be structural in a gas station, nonstructural in a highrise building. Finally, if a component does not contribute significantly to strength or stiffness, and is added by the owner or a vendor after the construction punchlist is completed, it is contents.

The method presented here estimates vulnerability based on the contribution to repair cost from only the top few components. By "top components" is meant the ones that contribute most to construction cost (new). Fragility functions for each category of component are taken from FEMA P-58 (2012), using a database that has been copied to Nexus, or they can be developed by the analyst using procedures specified for FEMA P-58. Fragility functions are defined in terms of the repairs required to restore the component to its pre-earthquake state, rather than in general qualitative terms. Repair costs for each component fragility function are taken from FEMA P-58, they can be developed by the analyst, or they can be taken from FEMA P-58 with an adjustment to account for local labor costs; the adjustment procedure is provided here. This is important: the FEMA P-58 material provides US-centric component repair costs, but it also provides guidance on how to use those repair costs to estimate component repair costs in other countries, using a factor that relates local construction labor costs to US labor costs. And the analyst is free to consult with a local construction contractor to estimate local repair costs for the top components and their damage states.

Our purpose here is to offer a procedure for estimating the vulnerability of a whole aggregate class highrise buildings, meaning all the structural and nonstructural components in a building as a whole, not every doorknob and exit sign. We also mean a probabilistic relationship between ground motion and repair cost as a fraction of replacement cost new of the entire building (not including contents).

The relationship is defined for an asset class that the user specifies using the GEM taxonomy. Even the basic GEM taxonomy is very flexible, with effectively unlimited combinations available. For illustration purposes, we assume that the asset class that is defined by three attributes: an occupancy class (residential, commercial, or industrial); a height range (low, mid, or highrise, which is actually just a proxy for a small-amplitude fundamental period of vibration); and a general category of lateral load resisting system (or more precisely a mode shape representing either an idealized frame, shearwall, or an intermediate case). The height range and lateral load resisting system can alternatively be replaced by calculated mode shape and 3-dimensionsal relationship between 5% damped elastic spectral acceleration response, roof absolute acceleration, and roof relative displacement (relative to the ground). For modern code-conforming structures, we expect that the lateral load resisting system will not make much difference, and that the vulnerability function will only depend on occupancy class and height range.

The methodology can be readily applied to finer occupancy categories—IT manufacturing facilities, for example—or to different combinations of taxonomic features, but we will not elaborate here. The adaption should be fairly obvious. In developing these guidelines, we have striven to achieve the following additional objectives:

- General applicability anywhere in the world.
- Requiring no greater skillset than a structural engineer with a master’s degree.
- Allowing for exercise of engineering judgment, but with defaults as guidance for less experienced analysts.
- Brevity (procedure defined in fewer than 50 pages).
- Flexibility (ability to accommodate various levels of effort).

2.2 Brief summary of the methodology

The methodology has five steps.

In Step 1, the analyst defines the asset class with one, three, or seven index buildings. An index building is a single particular specimen building of the asset class, with given geometry, materials, material properties, etc. If one index building is used, the analyst defines the asset class using a typical-quality case.

If the analyst wishes to explicitly quantify uncertainty with modest effort, then three index buildings are defined: a poor, typical, and superior-quality case. The poor-quality index building might be one with relatively low design base shear and relatively fragile or poorly anchored component types. The typical-quality index building might be one with typical (for the class) design base shear and a typical mixture of fragile and rugged component types. The superior-quality index building might be one with a high (for the class) design base shear and a relatively rugged mixture of components.

If the analyst wishes to quantify uncertainty to a high degree, including within-building uncertainty, then seven index buildings are defined that span at least three important dimensions that vary within the asset class. For example, for an asset class defined in terms of structural material, lateral load resisting system, height range, and occupancy, the top three uncertainties that vary within the class might be number of stories, vertical irregularity, and design base shear. The analyst is free to select different top uncertainties if the asset class fixes one or more of these.

Regardless of whether 1, 3, or 7 index buildings are used, each index building is assigned a particular number of stories, structural material per the GEM taxonomy, lateral load resisting system (LLRS) per the GEM taxonomy, a broad category of LLRS (shearwall, frame, or mixed), degree of plan irregularity (maximum ratio of long building wing length to shorter wing length), degree of vertical irregularity (maximum ratio of story height below to story height above), design base shear, and lastly, story by story, the quantity (in terms of replacement cost) of each of the top 1 or 2 structural component categories and top 5 or 6 nonstructural component categories. Omit the structural components if the analyst only wants to estimate non-structural vulnerability.

For the case of seven index buildings, three quality-level variants are defined for each index building. The variants differ in the selection of detailed component types. Since structural component types are largely defined by the index building, it is the detailed taxonomic categories of the nonstructural components that vary between variants. To explain, for each nonstructural component category defined at the level of the NISTIR 6389 component taxonomy (e.g., C1011 Fixed Partitions), there may be many choices for detailed component category, as defined at the level of FEMA P-58's component taxonomy. (For example, FEMA P-58 PACT database currently offers fragility functions and repair costs for 12 types of partitions, such as full-height and partial height gypsum board on metal stud, with wallpaper, ceramic tile, or stone finish. Some of these are relatively fragile, with low median drift capacity, some relatively rugged with median drift capacity 3 times that of the most fragile ones.)

In the poor-quality variant, nonstructural components are selected to reflect relatively vulnerable component types, such as unanchored equipment or relatively fragile partitions. In the typical-quality variant, nonstructural components are selected to reflect a typical mix of rugged and more-fragile component types. In the superior-quality variant, nonstructural components are selected to reflect a relatively rugged mix of component types, such as mostly well anchored equipment and relatively rugged structural and architectural components.

Step 2 is to derive two story-level vulnerability functions given non-collapse. One of the two is for drift-sensitive components and the other for acceleration-sensitive components. (The probability of collapse and the cost given collapse is addressed in a later step.) The vulnerability function is essentially the sum over all the components of the product of the probability of the component being in each damage state, the repair cost per unit of the component given the damage state, and the quantity of the components at that story. The story-level vulnerability function gives the mean damage factor of the components on that story, as a function of the peak drift or peak floor acceleration at that floor, given non-collapse.

Step 3 is to perform a structural analysis with the objective of estimating peak transient drift ratio at each story, peak floor acceleration at each floor, and collapse probability. All three measures (drift by story, acceleration by floor, and collapse probability) are evaluated at each of many intensity measure levels. The analyst is free to choose the intensity measure type of interest, but we offer default intensity measure types: for one index building, we suggest choosing the geometric-mean 5% damped elastic spectral acceleration response at 0.3 second period for lowrise (1-3 story) construction or 1.0 second period for taller buildings. For three or seven index buildings, we recommend the geometric mean of the 5% damped elastic spectral acceleration response at the small-amplitude fundamental period of vibration of each building.

We offer three options for structural analysis. The first assumes a mode shape appropriate either to a shearwall system, a frame system, or a mixed system. If shearwall system, the mode shape is calculated by first principles from the deformed shape of a prismatic cantilever column with infinite shear stiffness and finite bending stiffness and subjected to a triangular distributed lateral loading pattern. If a frame system, the mode shape is calculated by first principles from the deformed shape of a prismatic cantilever column with finite shear stiffness and infinite bending stiffness and subjected to a triangular loading pattern. If a mixed system, the model shape is triangular. Peak transient drifts and floor accelerations are then calculated based on the mode shape and spectral acceleration response of an equivalent single-degree-of-freedom harmonic oscillator with an elastic-perfectly plastic pushover curve. The second structural analysis option is to perform multiple nonlinear dynamic structural analyses of each index building. We offer guidance on selecting ground motions and performing structural analyses from FEMA P-58. The third structural analysis option is for the analyst to perform any type of structural analysis that is convenient or familiar, as long it estimates the structural response measures of interest at each of the intensity measure levels of interest.

Collapse is accounted for using the theorem of total probability: the damage factor at a given level of intensity is taken as the collapse probability at that intensity times unity (meaning that we assume a damage factor of 1.0 given collapse) plus the probability of non-collapse times the sum of the story-level damage factors given non-collapse. The collapse probability is quantified by defining a collapse capacity in terms of a lognormal cumulative distribution function of the intensity measure of choice. Default parameter values of both the pushover curve and default collapse capacity are taken from FEMA P-695 (Applied Technology Council 2009) and ASCE 7-10 (American Society of Civil Engineers 2010).

Step 4 is to derive a building-level vulnerability function by relating ground motion to story-level motion, calculating story-level loss, and summing over stories. The building-level vulnerability function is factored up to account for the fact that only the top 6 to 8 component categories are inventoried, and may not reflect all the value of the building. The theorem of total probability is then applied to include both the contribution from collapse and the contribution from repair given non-collapse.

The mean vulnerability function for the asset class is defined as the expected value of repair cost as a fraction of replacement cost new, conditioned on intensity measure level. If a single index building is used, the mean vulnerability function for the class is taken as the mean vulnerability function for the single, typical index building. If three index buildings are defined, the mean vulnerability function for the class is taken as an equally weighted average of the mean vulnerability functions for the three index buildings. If seven index buildings are used, the mean vulnerability function for the class is taken as a weighted average of the seven using weights derived to match the first several joint moments (mean, variance, skewness, etc.) of the top uncertainties, using a procedure called moment matching.

Uncertainty is quantified in terms of the coefficient of variation (COV) of the damage factor conditioned on the intensity measure level. If one index building is used, the COV is taken using a default function implied by HAZUS-MH. If three, COV is taken as 1.4 times the coefficient of variation implied by the three index buildings' mean vulnerability functions, that is, 1.4 times the building-to-building variability of mean damage factor. The factor of 1.4 reflects an assumption that within-building uncertainty (from record-to-record variability in ground motion, uncertain damage of building components, and uncertain costs to repair damage) is about equal to the building-to-building variability. If seven index buildings are used, COV is taken as a function of the within-building COVs, the seven mean vulnerability functions, and the moment-matching weight of each index building. An illustrative example is provided.

3 Literature Review

We reviewed existing literature related to the following topics:

- Dominant nonstructural component categories, that is, categories that tend to dominate nonstructural construction losses and nonstructural construction cost
- Dominant analytical procedures for modelling building component repair cost
- Collapse fragility, which can contribute to loss
- Uncertainty in seismic vulnerability

Dominant nonstructural categories. To begin, we reviewed reconnaissance reports from about 10 recent major earthquakes to identify the component categories that tend to dominate nonstructural losses, either because they are the most numerous kinds of nonstructural components, the most costly, the most fragile, or some combination. These dominant nonstructural components are generally the following:

- Interior partitions
- Exterior closure
- Ceilings
- Heating, ventilation, and air conditioning equipment
- Electrical equipment
- Plumbing equipment

However, we also found that detailed nonstructural elements vary substantially between building types and countries. It is therefore contingent on the analyst to know the asset class well enough to identify the top 5 or so categories of nonstructural component in terms of contribution to construction cost (new). Several sources provide additional guidance. In the United States, RS Means publishes square-foot construction cost manuals that estimate the cost to construct new buildings of several dozen models, where models are generally defined by occupancy, size, exterior wall material, and various options for finishes. Spons offers similar manuals for various countries around the world (e.g., Spons 2013). And local construction contractors possess local expertise that is not published in these manuals.

Dominant analytical procedures for modelling earthquake-induced repair cost. For analytical procedures, we mostly considered FEMA P-58 (ATC 2012). While probably too detailed for use here without modification, it represents the state of the art for estimating uncertain future building performance in terms of repair costs, life-safety impacts, and loss of functionality (dollars, deaths, and downtime). For the reader unfamiliar with FEMA P-58, it uses many MDOF nonlinear dynamic structural analyses to estimate uncertain structural response (member forces, deformations, and especially floor-by-floor acceleration and story-by-story peak transient interstory drift) at many intensity levels. Structural analysis is followed by a damage and loss analysis procedure that works at the component level. Building components are defined at a somewhat finer level of detail than NISTIR 6389 (National Institute of Standards and Technology 1999), itself a proposal to subdivide UNIFORMAT II. The FEMA P-58 taxonomy (adapted from Porter 2005) subdivides the NISTIR 6389 categories into subcategories that distinguish among seismic installation conditions, size, or both. The FEMA P-58 damage-analysis procedure depicts nonstructural damage in discrete damage states using fragility functions derived from experiment, earthquake experience, first principles, and in some cases expert judgment. Repair cost and repair duration are estimated based on construction experience data such as Xactimate or RS Means. FEMA P-58 offers a large though not exhaustive library of damage and repair models. It probably covers a majority of components that would be found in most classes of ordinary residential or commercial US buildings built in the last 50 years. The focus in the FEMA P-58 library is on structural and nonstructural components.

FEMA P-58's fragility database offers fragility functions for many though not all building component categories observed in real buildings around the world. The fragility database is large, too large to duplicate here. It is provided at the GEM Nexus website. Note again however that the FEMA P-58 fragility library generally offers fragility functions for 300 categories of structural components and 450 categories of nonstructural components. Structural components in the FEMA P-58 fragility library include various elements of steel, reinforced concrete, reinforced masonry, and timber construction (though not unreinforced masonry). Nonstructural components in the FEMA P-58 fragility library include various types of exterior closure, interior partitions, ceilings, floor coverings, and a wide variety of mechanical, electrical and plumbing components.

FEMA P-58 builds on a long history of prior art that we will not recap here, other than to refer the interested reader to seminal performance-based engineering work at the Massachusetts Institute of Technology by Czarnecki (1973), application by URS Corporation (Kustu et al. 1982), HAZUS-MH (e.g., Kircher and Whitman 1997), and 2nd-generation performance-based earthquake engineering development for the CUREE-Kajima Joint Research program and the Pacific Earthquake Engineering Research Center (Beck et al. 1999, Porter 2000, Porter et al. 2001, Porter 2003, and Goulet et al. 2007). Nor will we recap work to develop empirical nonstructural vulnerability functions, e.g., Porter et al. (2010). We note that FEMA P-58 offers standard procedures for deriving fragility functions, which are summarized in Porter et al. (2007).

Collapse fragility. Aslani and Miranda (2006) have shown that, at least in the case of an older nonductile reinforced concrete moment-frame building (the Van Nuys California Holiday Inn), collapse can contribute as much to expected annualized repair cost as does damage without collapse. This means a vulnerability function needs to account for collapse fragility. When considering collapse fragility, we reviewed FEMA P-695 (Applied Technology Council 2009), which like many other sources such as Luco et al. (2007) suggests that collapse fragility can be modelled with a lognormal cumulative distribution function using 5% damped elastic spectral acceleration response at the building's approximate fundamental period of vibration. Consistent with US standards (ASCE 2010), these authors explicitly assume the building responds with 100% of its reactive mass participating at the fundamental mode of vibration. FEMA P-695 in particular estimates median collapse capacity $\hat{S}_{CT}$ as the product of:

C_S Seismic response coefficient. C_S is essentially the design base shear normalized by building weight.

R Response modification factor. R is essentially a design ductility demand that approximates ductility capacity. According to ASCE 7-10, R varies between 1 and 8, and is tabulated here (in Appendix C) for a variety of materials and structural systems.

CMR Collapse margin ratio. *CMR* is essentially the ratio of collapse capacity to 1.5 times design ground motion, both in terms of $S_a(T_1,5\%)$. It is typically around 1.5 to 2.0.

SSF Spectral shape factor. *SSF* is a second-order modifier (i.e. second order compared with R and CMR) of roughly 1.15 that accounts for the difference between the shapes of real ground motions, which tend to be peaked, to design spectra.

The same work suggests that the uncertainty in collapse capacity can be modelled as the product of four unit-median lognormal distributions that account for record-to-record variability in ground motion, design requirements, test data, and modelling uncertainties. Without going into detail, the logarithmic standard deviation varies between approximately 0.5 and 0.9.

Loss uncertainty. Porter (2010) offers an estimate of the coefficient of variation of building damage factor as a function of mean damage factor, as implied by the HAZUS-MH vulnerability functions. The expression is $v(y) = 0.25y^{-0.5}$ where y denotes the mean damage factor (repair cost as a fraction of replacement cost new) and v denotes the coefficient of variation of damage factor. Data in ATC-13 (Applied Technology Council 1985) implies a relationship for residential or commercial "equipment" of $v(y) \approx 0.2y^{-0.25}$. Finally, an unpublished analysis of the COV for a class of buildings as implied by Porter (2010) and for individual, particular buildings as implied by Porter et al. (2005) suggests that the building-to-building variability of damage factor approximately equals the uncertainty in damage factor for an individual building. These facts will be useful when estimating the coefficient of variation of damage factor if too few index buildings are used to explicitly or completely estimate uncertainty.

4 Analytical Procedure

4.1 Overview of uncertainty propagation used here

This section provides an overview of the procedure used here to account for uncertainty in the seismic vulnerability function of a class of buildings. There are many ways to propagate uncertainty in seismic vulnerability functions for classes of buildings: Monte Carlo simulation, Latin Hypercube simulation, moment matching, and probably others. One could try to propagate all uncertainties at every stage in each sample building. These guidelines propose procedures with low, medium, or high level of effort.

The low-effort procedure requires no uncertainty propagation by the analyst: a single index building that the analyst considers to be typical of the class is modelled. Its mean damage factor is calculated, and this is taken as the mean damage factor of the class. A coefficient of variation of damage factor for the class is assumed, taken from a prior study of uncertainty in HAZUS-MH building classes (Porter 2010). See Figure 1a.

The medium-effort procedure requires the analyst to model poor, typical, and superior-quality index buildings that represent the analyst's opinion of the range of performance of the class. The simple average of their mean damage factors is taken as the mean damage factor for the class. The sample coefficient of variation of the three index buildings is taken as the between-building variability of the vulnerability of the class. The analyst assumes that the within-building variability approximately equals the between-building variability (an assumption supported by limited testing), and that the variance of the two can be summed to calculate the variance of the vulnerability function for the class, which equates with multiplying the sample COV by 1.41, as shown in Figure 1b.

The high-effort procedure is called moment matching; see Cho et al. (2013) in Appendix A for details. It is a generalization of Gaussian quadrature that is commonly used in finite element analysis. Briefly, the analyst selects the three leading factors or variables that influence the vulnerability of the class. The analyst calculates their joint probability distribution from a large sample of buildings in the class, and then selects seven sets of all three variables. Each sample has a fixed value of each of the three factors, and an associated weight for the sample. The samples and weights are selected so that the first five moments (mean, variance, skewness, etc.) of the weighted sample set match the first five moments of the population as a whole. When the weighted average of the vulnerability functions of the seven index buildings is calculated, it will estimate the mean damage factor of the class with 5th order accuracy, that is, like a Taylor Series expansion to the 5th term. Between-building variance (i.e., the uncertainty in the class vulnerability function associate with building-to-building variability) is accurate to 2nd order. Within-building variability (the uncertainty in the vulnerability of a given index building) is calculated through Monte Carlo simulation of the ground-motion time history, damage, and loss. The mean damage factor and coefficient of variation of damage factor is calculated for each of the seven index buildings. The weighted combination of the probability density functions of the seven index buildings is integrated to calculate the coefficient of variation of the vulnerability function of the class, considering both within-building and between-building variability. See Figure 1(c) for illustration and Appendix A for theoretical justification. Note that moment matching does not require that the joint distribution of the variables be continuous, unimodal, or that the variables be independent. The present work is limited to variables that are independent, but Ching et al. (2008) show how to rotate axes to account for dependence between the variables.

A reasonable alternative to moment matching is to use a procedure called class partitioning. See Diday et al. (2005) for detail on this approach. Class partitioning is briefly summarized in Appendix D, but is not illustrated here.

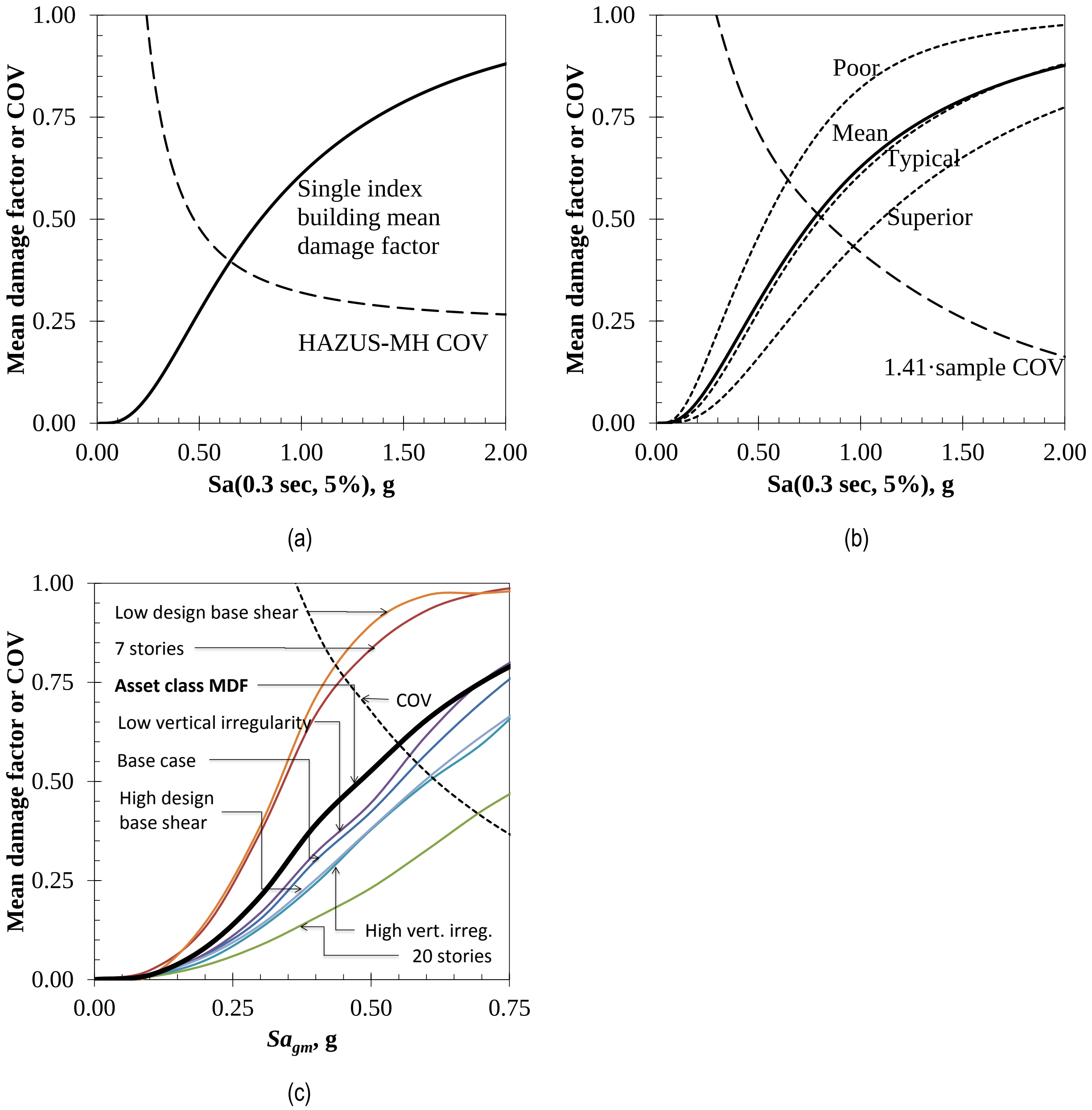


Figure 1. Uncertainty propagation for (a) 1, (b) 3 and (c) 7 index buildings

4.2 Step 1: Define the asset class at risk with one, three, or seven index buildings

This procedure consists of four steps, beginning with definition of the asset class via a few attributes of either one, three, or seven index buildings.

How many index buildings? If the user is constrained regarding time and does not mind treating uncertainty with a default relationship, use a single index building that represents a median or typical case, as described in more detail shortly. With more time, one can explicitly propagate uncertainty. The user can select the characteristics and nonstructural inventory for three index buildings: one a poor case, with relatively fragile components, a typical case like the one just mentioned, and a superior case, with relatively rugged or seismically restrained components. Some judgment is required to establish the characteristics of these variants. Finally, with seven index buildings and some additional Monte Carlo simulation, one can explicitly propagate uncertainty associated within and between specimens. Again, details are presented shortly.

One index building. Select the taxonomy attributes of the asset class of interest (E.g., lowrise, midrise, or highrise building? Reinforced concrete, steel, or masonry structural material? Residential, commercial, or industrial occupancy? And so on. The user must decide which attributes to use to define the asset class; we provide no guidance on that point.) Create an inventory of building components for a single index building, that is, for a typical case, with a mix of components that are neither much more rugged than typical nor very fragile. By "component" we mean a category of parts of a building as defined by the FEMA P-58 taxonomy (an extension of NISTIR 6389, itself an extension of UNIFORMAT II). See the FEMA P-58 fragility database at GEM Nexus for the taxonomy. Each line of the fragility tab in the spreadsheet represents a component category.

The analyst will see in the FEMA P-58 spreadsheet that most component categories have multiple variants, some more fragile than others, some less. The median capacity of the first damage state is an indicator of the relative fragility of a component. By "inventory" is meant the quantities of the components of the building on a story-by-story basis. Use the FEMA P-58 taxonomy or any other convenient taxonomy where the replacement cost and fragility of each class of component can be estimated. For present purposes, these guidelines adopt the FEMA P-58 component taxonomy by reference. The analyst should seek the latest version of the fragility database from the Applied Technology Council or US Federal Emergency Management Agency, since the present authors and GEM may not update the FEMA P-58 spreadsheet on GEM Nexus. To create an inventory, document the design of the index building as shown in Table 1 (overall description of the index building), Table 2 (identify top 6 components), Table 1 (fragility functions and unit repair costs for top 6 components), and Table 5.

Three index buildings. With more time or need to explicitly propagate uncertainty, the analyst can select the characteristics and inventory for three index buildings: one a poor case, with relatively low design base shear and fragile components, a typical case like the one just mentioned, and a superior case, with relatively rugged or seismically restrained components and relatively high design base shear. Other variables to consider to differentiate poor, typical and superior quality index buildings include number of stories, presence of vertical irregularity, and presence of plan irregularity. The poor, typical, and superior-quality index buildings should represent in the analyst's mind cases where repair cost would be exceeded respectively by 10%, 50%, and 90% of buildings of similar occupancy. This approach will quantify variability of vulnerability between specimens in the asset class. The uncertainty in vulnerability within an individual specimen will be assumed to be equal to the between-specimen variability. Document the design of each index building with one set of tables, Table 1 (overall description of the index building), Table 2 (identify top 6 nonstructural components), Table 3 (identify top 2 structural components), Table 4a-h (fragility functions and unit repair costs for top 6 nonstructural components and top 2 structural components), and Table 5 (component inventory by story). That is, complete Table 1 through Table 5 for each index building.

Seven index buildings. Finally, the analyst can explicitly quantify within-specimen variability using Monte Carlo simulation and more thoroughly quantify the between-specimen variability using a procedure called moment matching that we specify here. (A reasonable alternative approach to quantifying between-specimen variability is to use a procedure called class partitioning, but we do not specify that procedure here.) This approach requires advanced skills in simulation and some skills to implement moment matching. The necessary probability information is provided here, but not details of the Monte Carlo simulation. If the analyst does not already possess the required skills, this approach is not recommended. This approach requires the analyst to quantify the probability distribution within the asset class of three key attributes: number of stories, degree of vertical irregularity, and design base shear. (The analyst is free to choose other features that he or she believes more strongly influence variability between specimens in an asset class, such as degree of plan irregularity, but the analyst will need to quantify the probability distribution of each such feature.) All these probability distributions must be estimated or compiled by observation of many samples. The procedures for making the necessary observations and selecting the values of the key attributes are detailed in the manuscript in Appendix A. The tables for defining the samples of the asset class are provided in Appendix B, along with notes defining number of stories, degree of plan irregularity, and degree of vertical irregularity, as well as notes about probability distributions for structural analysis, damage analysis, and loss analysis. The reader may wonder why not use Monte Carlo Simulation throughout, including in the design of index buildings? The reason is pragmatic: to create a structural model is time consuming compared with the rest of the analysis procedure proposed here. Moment matching allows the analyst to create a small number of structural models that sample over a few features that matter most to structural response, far more efficiently than does Monte Carlo simulation. See Ching et al. (2008) and Cho et al. (2013) for details. Again, class partitioning, as summarized in Appendix D and detailed in Diday et al. (2005) is a reasonable alternative.

Table 1. Index building definition (1 or 3 index buildings)

Asset class (e.g., material, LLRS, height category, occupancy)	
Structural material (if used, from GEM taxonomy)	
Lateral load resisting system (if used, from GEM taxonomy)	
Broad category (choose one)	Frame, shearwall, mixed
Height category (if used, from GEM taxonomy)	
Occupancy (if used, from GEM taxonomy)	
Other attribute 1	
Other attribute 2	
Other attribute 3	
Index building quality (if using 3 index buildings; choose one)	Poor, typical, or superior
Index building name (if a particular real building is selected)	
Index building model (if using local per-square-meter cost manual)	
Stories	
Story height (m)	
Building height (m)	
Design year	
Construction year	
Labor cost as a fraction of total labor + material in construction cost	
Local labor cost as a fraction of US labor cost	
Small amplitude fundamental period of vibration T, sec	
Gamma (default = 1.3)	
Median collapse capacity, $\hat{S}_{CT}$, g, IM = Sa(T, 5%), geomean	
Logarithmic standard deviation of collapse capacity (default 0.8)	
Design base shear as fraction of building weight C_S	
Cost manual reference (if used)	
Total building cost (currency per m^2)	
Total building floor area (m^2)	
Total building construction cost (currency)	
Fraction f_1 inventory construction cost as fraction of RCN	

Table 2. Ranking of nonstructural components in decreasing order of contribution to construction cost (1 or 3 index buildings)

Rank	1	2	3	4	5	6
Description						
NISTIR 6389 class ID						
FEMA P-58 class ID						
Demand param (PFA or PTD)						

Table 3. Ranking of structural components in decreasing order of contribution to construction cost (1 or 3 index buildings)

Rank	1	2
Description		
NISTIR 6389 class ID		
FEMA P-58 class ID		
Demand param (PFA or PTD)		

Table 4a. Fragility functions and unit repair costs, nonstructural component rank 1 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #1				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 4b. Fragility functions and unit repair costs, nonstructural component rank 2 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #2				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 4c. Fragility functions and unit repair costs, nonstructural component rank 3 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #3				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 4d. Fragility functions and unit repair costs, nonstructural component rank 4 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #4				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 4e. Fragility functions and unit repair costs, nonstructural component rank 5 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #5				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 4f. Fragility functions and unit repair costs, nonstructural component rank 6 (1 or 3 index buildings)

Nonstructural Component Specification, Rank #6				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 4g. Fragility functions and unit repair costs, structural component rank 1 (1 or 3 index buildings)

Structural Component Specification, Rank #1				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 4h. Fragility functions and unit repair costs, structural component rank 2 (1 or 3 index buildings)

Structural Component Specification, Rank #2				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 5. Component inventory by story (1 or 3 index buildings)

	Nonstructural components						Structural components	
Rank:	1	2	3	4	5	6	1	2
Name								
Unit								
Story	Quantity (total)							
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								

Add rows as necessary

4.3 Step 2: Derive story-level vulnerability functions

This step is described in detail only for the cases of 1 and 3 index buildings. It is assumed that only advanced analysts will examine 7 index buildings using Monte Carlo simulation (MCS), and that these advanced users can infer the necessary MCS tasks from the following. In this step, one calculates the story-level vulnerability function for all acceleration-sensitive components on each story, and all drift-sensitive components on each story. The vulnerability function is evaluated at several levels of floor acceleration. Let

i_a = index to acceleration-sensitive component types, $i \in \{1, 2, \ldots N_a\}$

i_r = index to drift-sensitive component types, $i \in \{1, 2, \ldots N_r\}$

N_a = number of acceleration-sensitive component types included in the inventory

N_r = number of drift-sensitive component types included in the inventory

s_{ha} = peak floor acceleration at floor h, to be evaluated at $s_{ha} \in \{0, 0.01, 0.02, \ldots 3.00\}$

s_{hr} = peak transient drift at story h, to be evaluated at $s_{hr} \in \{0, 0.005, 0.010, \ldots 0.20\}$ (the drifts reach 0.20 because woodframe buildings can tolerate drifts in excess of 10% without collapse)

$Ca_{h,ia}(s)$ = (uncertain) repair cost of acceleration-sensitive components of type i_a at floor h

$Cr_{h,ir}(s)$ = (uncertain) repair cost of acceleration-sensitive components of type i_r at floor h

E[$Ca_{h,ia}(s)$] = expected value of repair cost of acceleration-sensitive components of type i at floor h

E[$Cr_{h,ir}(s)$] = expected value of repair cost of drift-sensitive components of type i at floor h

Evaluating E[$Ca_{h,ia}(s)$] and E[$Cr_{h,ir}(s)$] involves the fragility functions and unit repair costs for each fragility function. Here, it is assumed that all fragility functions are in the form of lognormal cumulative distribution functions. It is also assumed that all components with multiple damage states in addition to the undamaged state have sequential fragility functions, that is, a component must pass through damage state d before it can reach damage state d+1. There are also cases of components with either simultaneous or mutually exclusive and collectively exhaustive (MECE) damage states. For treating these, see Box 1. Note that there is frequent use here of mean, median, standard deviation, logarithmic standard deviation, and coefficient of variation. See Box 2 for conversions among these parameters.

Box 1: components with simultaneous or MECE damage states

There are cases of components with either simultaneous or mutually exclusive and collectively exhaustive (MECE) damage states. In the FEMA P-58 fragility library, elevators are an example of the first case, steel beams are an example of the second. These have a single fragility function for the occurrence of any damage, and a probability mass function (PMF) for the alternative damage states. In the case of simultaneous, the probabilities in the PMF sum to a value greater than 1.0. In the case of MECE damage states, they sum to 1.0. To deal with either case, we simplify the loss analysis by assigning an expected value of unit repair cost (cost to restore a unit of the component type to the undamaged state) and the coefficient of variation of unit repair cost as shown in Equation (1) and (2), respectively. Again, these two equations are only needed in the case of simultaneous or MECE damage states. In these equations, $m_{i,1}$ and $v_{i,1}$ respectively denote the mean and coefficient of variation of repair cost for a unit of component of category i given that any damage occurs; d denotes the possible damage states, N_D denotes the number of possible damage states; $p_{i,d}$ denotes the probability that a component of category i, once damaged, is in damage state d; $\dot{m}_{i,d}$ denoted the expected value of repair cost given that a component of category i, once damaged, is in damage state d; and $\sigma_{i,d}$ denotes the standard deviation of repair cost given that a component of category i, once damaged, is in damage state d. Henceforth the component with simultaneous or MECE damage states can be treated like one with a single possible damage state, i.e., N_D = 1.

$$m_{i,1} = \sum_{d=1}^{N_D} p_{i,d} \cdot \dot{m}_{i,d} \tag{1}$$

$$v_{i,1} = \frac{1}{m_{i,1}} \sqrt{\sum_{d=1}^{N_D} p_{i,d} \cdot \left(\sigma_{i,d}^2 + \left(\dot{m}_{i,d} - m_{i,1} \right)^2 \right)} \tag{2}$$

There are a few components in the FEMA P-58 database that have a more-complicated set of damage states, e.g., several sequential damage states in which one is a MECE combination of two or more finer damage states. These combinations can be collapsed so that their repair cost can be represented as if the combination were a single equivalent sequential damage state, using Equations (1) and (2). See Box 3 for additional guidance on estimating local repair costs.

In the case of lognormal fragility functions and sequential damage states (or simultaneous or MECE treated as described in Box 1), the mean fraction of specimens of type *i* that are damaged in damage state *d* is the expected per-specimen failure probability (denoted here by $p_{i,d}$) and given by Equation (3).

$$\begin{aligned} p_{i,d}(s) &= \Phi\left(\frac{\ln\left(s/\theta'_{i,d} \right)}{\beta'_{i,d}} \right) - \Phi\left(\frac{\ln\left(s/\theta'_{i,d+1} \right)}{\beta'_{i,d+1}} \right) && d < N_D \\ &= \Phi\left(\frac{\ln\left(s/\theta'_{i,d} \right)}{\beta'_{i,d}} \right) && d = N_D \\ &\geq 0 \end{aligned} \tag{3}$$

where

$$\beta'_{i,d} = \sqrt{\beta_{i,d}^2 + \beta_m^2} \tag{4}$$

$$\theta'_{i,d} = \theta_{i,d} \cdot \exp\left(1.28 \cdot \left(\beta'_{i,d} - \beta_{i,d} \right) \right) \tag{5}$$

In Equation (3), *s* denotes peak floor acceleration in the case of acceleration-sensitive components and peak transient drift in the case of drift-sensitive components; $\theta_{i,d}$ and $\beta_{i,d}$ are median and logarithmic standard deviation of capacity of the fragility function for component category *i*, damage state *d*, taken from Table 4a through Table 4f, and N_D is the number of damage states in addition to the undamaged state. The floor of zero in Equation (3) is included to allow for different values of β in the denominators, which can lead to fragility functions that cross. The parameter β_m in Equation (4) adds uncertainty associated with approximations in the structural model. In the case of a pushover structural analysis, β_m accounts for the approximation of using a pushover rather than multiple nonlinear dynamic analyses (β_m = 0.3, from FEMA P-695 [Applied Technology Councl 2009]). If the analyst employs nonlinear dynamic structural analysis, take β_m = 0 as in FEMA P-58. Supporting documentation regarding the errors inherent in nonlinear static pushover based methods of analysis can be be found in NIST GCR 10-917-9 (National Institute of Standards and Technology, 2010).

$$\begin{aligned} \beta_m &= 0.4 && \text{simplified} \\ &= 0.3 && \text{nonlinear static} \\ &= 0 && \text{nonlinear dynamic} \end{aligned} \tag{6}$$

Because increasing uncertainty with $\beta_m > 0$ will tend to bias risk upwards, $\theta_{i,d}$ is adjusted to ensure that the fragility function with the higher β is rotated about the 20th percentile rather than the 50th, that is, the excitation associated with 20% failure probability is unchanged, hence the modification in Equation (5).

The mean vulnerability function per component of category *i* is given by

$$E\left[C_i \middle| S_{h,i} = s \right] = N_i \cdot \sum_{d=1}^{N_D} p_{i,d}(s) \cdot m_{i,d} \tag{7}$$

where $m_{i,d}$ denotes the mean repair cost per unit of component category i, damage state d. It can be calculated from the median unit repair cost P_{50} and logarithmic standard deviation of unit repair cost b, as follows.

$$m_{i,d} = P_{50,i,d} \exp\left(0.5 \cdot b^2\right) \tag{8}$$

See Box 2 for additional guidance on estimating $m_{i,d}$. Now estimate the story-level vulnerability for acceleration sensitive components as

$$E\left[C_a \middle| S_{h,a} = s_a\right] = \sum_{i=1}^{N_a} E\left[C_i \middle| S_{h,a} = s_a\right] \tag{9}$$

Where C_a denotes the repair cost for acceleration-sensitive components. Repeat for drift-sensitive components:

$$E\left[C_d \middle| S_{h,d} = s_d\right] = \sum_{i=1}^{N_d} E\left[C_i \middle| S_{h,d} = s_d\right] \tag{10}$$

Where

C_a = repair cost of acceleration-sensitive components on the given story

C_d = repair cost of drift-sensitive components on the given story

E[X|Y] = expected (mean) value of X given conditions Y

$S_{h,a}$ = uncertain story-level acceleration at floor h

s_a = a particular value of $S_{h,a}$

$S_{h,d}$ = uncertain story-level drift ration (unitless) at story h

S_d = a particular value of $S_{h,d}$

i = an index to the component categories present on the story

N_a = number of acceleration-sensitive component categories present on the story

N_d = number of drift-sensitive component categories present on the story

$N_{i,h}$ = quantity of components of category i on story h, from Table 5.

Φ = standard normal cumulative distribution function evaluated at the term in parentheses

θ_i = median capacity of the component category, from Table 4a-f.

β_i = logarithmic standard deviation of the capacity of the component category from Table 4a-f.

Evaluate the mean story-level vulnerability function at floor accelerations $s \in$ {0.01, 0.02, … 3.0g} and drifts s $\in$ {0.001, 0.02, … 0.10} for each story h of each index building. These calculations can be done in a spreadsheet, without structural analysis.

Box 2: mean, median, standard deviation, logarithmic standard deviation, and coefficient of variation

Here are some useful conversions among the parameters of lognormally distributed random variables. Let

m = mean (same as "expected value")

θ = median (same as "50th percentile")

σ = standard deviation

β = logarithmic standard deviation

v = coefficient of variation

$$m = \theta \cdot \exp\left(0.5 \cdot \beta^2\right) \tag{11}$$

$$\theta = \frac{m}{\sqrt{1+v^2}} \tag{12}$$

$$\sigma = m \cdot \sqrt{\exp\left(\beta^2\right)-1} \tag{13}$$

$$v = \sqrt{\exp\left(\beta^2\right)-1} \tag{14}$$

$$\beta = \sqrt{\ln\left(1+v^2\right)} \tag{15}$$

Box 3: local repair costs

Note the requirement for unit repair costs $m_{i,d}$ expressed in the currency and local costs of the country of interest. The analyst can seek this information from local construction contractors, using the description of the repair efforts in associated with the damage state, such as from the FEMA P-58 PACT database. In a few cases, resources such as Xactimate (Xactware 2012) are available to provide local cost information. In case the analyst has no access to resource other than FEMA P-58 PACT database, we offer the following very approximate approach to estimating unit repair costs. Let

r_{lab} = ratio of hourly cost for local construction labor to the hourly cost for US construction labor. Estimate the hourly cost of US construction cost labor by referring to the US Bureau of Labor Statistics' cost data for construction laborers at http://www.bls.gov/oes/current/oes472061.htm. Other countries may have similar national data to estimate the hourly cost of local construction labor. If not, refer to a local construction contractor.

f_{lab} = fraction of US unit repair cost associated with labor. Absent better information, we recommend these default values:

= 0.9 efforts requiring repairing but not replacing architectural and structural components, mechanical, electrical, or plumbing equipment

= 0.5 efforts requiring replacing architectural and structural components

= 0.1 efforts requiring replacing mechanical, electrical, and plumbing equipment

$m_{i,d(US)}$ = mean unit repair cost in the US, e.g., according to the FEMA P-58 PACT database, available at GEM Nexus.

$m_{i,d}$ = mean unit repair cost in the local context

$$m_{i|d} = m_{i,d(US)}\left(1-f_{lab}\right)+m_{i,d(US)}\left(f_{lab}\cdot r_{lab}\right) \tag{16}$$

$P_{90(US)}$ = 90th percentile of unit repair cost in the US, from FEMA P-58 PACT database

$P_{10(US)}$ = 10th percentile of unit repair cost in the US, from FEMA P-58 PACT database

$\sigma_{i,d(US)}$ = standard deviation of unit repair cost in the US

$$\sigma_{i,d(US)} = \frac{1}{2.56}\left(P_{90}-P_{10}\right) \tag{17}$$

$\sigma_{i,d}$ = standard deviation of unit repair cost in the local context

$$\sigma_{i,d} = \sigma_{i,d(US)} \cdot \frac{m_{i,d}}{m_{i,d(US)}} \tag{18}$$

4.4 Step 3: Structural analysis

4.4.1 General requirements for structural analysis

The analytical estimation of seismic losses is based on combining probabilistic seismic hazard analysis with seismic vulnerability functions. This document focuses on the latter, but it is important to ensure that these two complex calculations mesh well. The point of contact between them is an interface variable that links seismic hazard with structural response. The variable is referred to here as the intensity measure (IM), referring to a measure of the intensity of ground motion. A rather unobvious requirement for the IM is that it needs to crystallize all the seismological properties of ground motion to make any assessment (a) practical (b) efficient and (c) sufficient with respect to the underlying issues related to site and source (Luco and Cornell 2007). Practicality necessitates the use of IMs for which ground motion prediction equations (GMPEs, also known as attenuation relationships) are available. This currently restricts the analyst to a choice among peak ground acceleration, peak ground velocity, peak ground displacement. and spectral acceleration response. The latter is where most research is currently focused among engineering seismologists. A more efficient IM is one that produces a lower uncertainty in structural response conditioned on seismic source and path parameter values in a fixed and ideally small number of time-history structural analyses. A sufficient IM is one that accounts for all source and path parameters that strongly affect structural response, so that conditioned on IM, response is independent of other source and path parameters. An IM that leaves the structural response sensitive to other parameters can cause unwanted bias to creep into loss estimates wherever the ground motion characteristics do not match the source and site requirements for the building and IM level that is being considered.

Together with the selection of IM comes the selection and scaling of ground motions for use in time-history structural analysis. Since vulnerability functions are needed for a large range of IM values, a structural model needs to be subjected to a wide range of IM values that will force it to show its full range of response and loss, from elastic response to global collapse. Because of limitations in the catalogue of ground motion recordings, it is often desirable to be able to modify (i.e. scale) a record to display the desired IM level. (Using artificial accelerograms is another potential approach, but it is not recommended for general use at this point unless the analyst possesses the necessary expertise). A sufficient IM theoretically allows unrestricted scaling of ground motions to match any IM level. In reality, though, no single IM is perfect. Therefore, exercising at least a minimum of care in selecting ground motions is advised. Since vulnerability functions are usually developed to be applicable to wide geographic regions, it is often not possible to select ground motion time histories according to the most recent research findings, using e.g. the conditional mean spectrum (Baker and Cornell, 2008), or incorporating near-source directivity. In general, our recommendation is to use the best possible IM that will allow a wide range of scaling, plus a suite of relatively strong ground-motion records recorded on firm soil. Whenever sufficient information exists about the dominant seismic mechanism or site soil in the region for which the vulnerability curve is developed (e.g. crustal earthquakes in western USA or soft soil in Mexico City), it may help an experienced analyst in choosing at least some of the characteristics of the records to use.

Being aware of the requirements, let us now consider the choice of IM. The most popular option is $S_a(T_1, 5\%)$, i.e. the 5% damped elastic spectral acceleration response at the period of interest (usually a structure's small-amplitude fundamental period of vibration, T_1). $S_a(T_1, 5\%)$ has been found to be both efficient and sufficient for first-mode-dominated, moderate period structures when directivity is not present, but has often been criticized for lack in sufficiency wherever large scale factors (higher than, say 3.0) are employed. This is mainly the case for modern structures that need considerably intense ground motions to experience collapse. On the other hand, this is rarely the case for older and deficient buildings. A good alternative for single buildings is $S_{agm}(T_i)$, defined here as the geometric mean of the 5% damped elastic spectral acceleration response at the estimated small-amplitude fundamental periods of the index buildings i. Use of this IM significantly improves the efficiency and the sufficiency of the estimation. It also remains practical as a GMPE for $S_{agm}(T_i)$ is easily estimated from existing GMPEs. It offers a considerable extension to the applicability of scaling (Vamvatsikos and Cornell, 2005a; Bianchini et al. 2010), and is the recommended approach whenever undertaking nonlinear dynamic structural analysis (e.g., IDA). Simpler, nonlinear static procedure methods are based on $S_a(T_1, 5\%)$ by default as a single-degree-of-freedom approximation lies at their basis. This cannot be changed and it is an often disregarded constraint of the pushover analysis. Its implications are simply accumulated together with the other (and often larger) errors that the approximate nature of the pushover incurs. It is doubtful whether the benefits of $S_{agm}(T_i)$ can be realized here, thus $S_a(T)$ at a common period T will be the recommended choice for parameterizing the vulnerability curves from such methods.

We offer a simplified default structural analysis procedure in Section 4.4.2. We offer guidance for nonlinear pseudostatic (pushover) structural analysis in Section 4.4.3 and for incremental dynamic analysis in Section 4.4.4. Or the analyst can use any other familiar structural analytical procedure options for performing the structural analysis. The objective of the structural analysis is to estimate geometric mean peak floor acceleration at each floor of the building, peak transient drift ratio at each story of the building, and probability of collapse, all three measures at each of many intensity measure levels.

x = intensity measure level. Select the intensity measure is most of interest. The default used here is as follows:

= S_a(0.3 sec, 5%) for index buildings with $T_1 \leq 0.5$ sec

= S_a(1.0 sec, 5%) for index buildings with 0.5 sec < $T_1 \leq 2.0$ sec

= S_a(3.0 sec, 5%) for index buildings with 2.0 sec < T_1

= $S_a(T_{1m}$, 5%) for index buildings analysed using nonlinear dynamic or static analysis with scaling factors < 3.0,

= S_{agm} for index buildings analysed using nonlinear dynamic analysis regardless of scaling level

$S_{agm} = \left[\prod_{i=1}^{n} S_a(T_i, 5\%) \right]^{1/n}$, i.e., the geometric mean of the 5% damped spectral acceleration response at *n* periods of vibration, T_i ($i = 1,\ldots n$)

Suggested T_i values for highrise buildings are the following five: {T_{2m}, $min(1.5 \cdot T_{2m}, 0.5 \cdot (T_{2m}+T_{1m}))$, T_{1m}, $1.5 \cdot T_{1m}$, $2 \cdot T_{1m}$}, where T_{1m} and T_{2m} denote the central value (e.g., average or median) of the first and second mode period, respectively, of the index building set. For use with lowrise buildings, where the second mode is not influential, only three T_i values are needed: {T_{1m}, $1.5 \cdot T_{1m}$, $2 \cdot T_{1m}$},Here, each measure of damped elastic spectral acceleration response S_a is the geometric mean of two orthogonal directions. For the default intensity measure types, evaluate structural response at the following intensity measure levels:

$T_{1m} \leq 0.5$ sec: x ∈ {0.1g, 0.2g, 0.3g, … 3.5g}
0.5 < $T_{1m} \leq 2$ sec: x ∈ {0.1g, 0.2g, 0.3g, … 1.5g}> 0.5
0.2 sec < T_{1m}: x ∈ {0.1g, 0.2g, 0.3g, … 0.5g}

If an index building is too computationally demanding to analyse at this many levels of x, the analyst can use

$x \in \{10^{-1.00}, 10^{-0.75}, 10^{-0.50}, \ldots 10^{0.50}g\}$. This requires analysis at a maximum of 7 intensity measure levels.

4.4.2 Option 1: simplified structural analysis

This method tends to over-predict acceleration response at the top of the building and to underpredict acceleration response at the bottom of the building, because it neglects higher modes. These errors should tend to cancel each other out when summing up the costs of different stories. Therefore, for simplicity, let

T_{1m} = mean fundamental period of vibration, sec., of the index buildings

$$T_{1m} = \frac{1}{n}\sum_{i=1}^{n} T_{1,i} \tag{18}$$

$T_{1,i}$ = estimated fundamental period of vibration of index building *i*, from structural analysis, local guidelines, or use defaults from ASCE 7-10:

= $0.0724Z^{0.8}$ steel moment-resisting frame

= $0.0466Z^{0.9}$ concrete moment-resisting frame

= $0.0731Z^{0.75}$ steel eccentrically braced frame or steel buckling-restrained braced frame

= $0.0488Z^{0.75}$ all others

Z = building height, meters

Let $s_{ha}(x)$ denote the story-level peak horizontal acceleration at floor *h* given that $S_a(T_{1m}$,5%) takes on a value of *x*. It can be calculated during the structural analysis, or approximated as follows:

$$s_{ha}\left(x\right)=PGA+\varphi\left(h\right)\cdot\left(\Gamma\cdot S_{a}\left(T_{1m},5\%\right)-PGA\right)$$
$$\leq S_{\max} \tag{19}$$

$$s_{hd}\left(x\right)=\Gamma\frac{S_{a}\left(T_{1m},5\%\right)T_{1m}{}^{2}}{4\pi^{2}}\cdot\frac{\left(\varphi\left(h+1\right)-\varphi\left(h\right)\right)}{z_{h+1}-z_{h}} \tag{20}$$

Where

PGA = peak ground acceleration in units of g

≈ S_a(1 sec, 5%) for $T_{1m} \geq 0.5$ sec

≈ $0.4\cdot S_a$(0.3 sec, 5%) for $T_{1m} < 0.5$ sec

φ(*h,x*) = response at floor h, normalized by response at the roof, given x. Evaluate it at each value of x through structural analysis. Alternatively, use the following default for all values of x as follows:

$$\varphi\left(h\right)=\frac{z^{2}}{Z^{5}}\cdot\frac{\left(70Z^{3}-40Z^{2}z+5Zz^{2}+2z^{3}\right)}{37} \qquad \text{shearwall building} \tag{21}$$

$$\varphi\left(h\right)=\frac{z}{Z^{3}}\cdot\frac{\left(12Z^{2}-3Zz-2z^{2}\right)}{7} \qquad \text{frame building} \tag{22}$$

$$\varphi\left(h\right)=\frac{z}{Z} \qquad \text{dual system} \tag{23}$$

(These default mode shapes for shearwall and frame buildings are derived from first principles using the deflected shape of a cantilever beam subjected to linearly increasing (triangular) distributed load. For a shearwall building, the beam is taken as having infinite shear modulus (GA = ∞) and finite bending stiffness (EI < ∞). For a frame building, the beam is taken as having finite shear modulus (GA < ∞) and infinite bending stiffness (EI = ∞). The default mode shape for a dual system is taken as linear.)

Γ = roof acceleration as a factor of modal acceleration

≈ 1.3

S_{max} = maximum strength of the fully-yielded system normalized by the effective seismic weight, *W*, of the structure, i.e., the y-value of the building's pushover curve at ultimate in spectral coordinates.

z = height of story *h* above the ground, meters

Z = roof height, meters above the ground

Calculate $s_{ha}(x)$ for each story *h* at each level of *x*.

Also calculate the collapse probability at each level of *x*, given by

$$P_{c}\left(x\right)=\Phi\left(\frac{\ln\left(x/\hat{S}_{CT}\right)}{\beta_{TOT}}\right) \tag{24}$$

where

$\hat{S}_{CT}$ = median collapse capacity of the building in terms of S_a(T,5%) and β_{TOT} denotes the total logarithmic standard deviation of the collapse capacity. The user can calculate both values by methods specified in D'Ayala and Meslem (2012), or by FEMA P-695, or by the following simplified method based on FEMA P-695, illustrated in Figure 2:

$$\hat{S}_{CT}=C_{S}\cdot1.5\cdot R\cdot CMR\cdot SSF \tag{25}$$

$$\beta_{TOT}=0.8 \tag{26}$$

C_S = seismic response coefficient (see below for example of how to calculate C_S in the United States). Note that it is defined as $C_s = V/W$, i.e., design base shear as a fraction of building weight, but in the US is calculated as shown later. It may be calculated in other ways in other countries.

V = design base shear, units of force

W = building weight

R = response modification factor, essential ductility demand at design-level ground motion. Can be taken from ASCE 7-10 Table 12.2-1 (duplicated in Appendix C) or from local standards.

CMR = collapse margin ratio, as defined in FEMA P-695: "The ratio of the median 5%-damped spectral acceleration of the collapse level ground motions, $\hat{S}_{CT}$ (or corresponding displacement, SD_{CT}), to the 5%-damped spectral acceleration of the MCE ground motions, S_{MT} (or corresponding displacement, SD_{MT}), at the fundamental period of the seismic-force-resisting system." In the US, ordinary buildings are designed to resist base shear of $2/3 \cdot S_{MT}/(R/I_e)$, where I_e is an importance factor, generally though not always 1.0, and R is as defined above. Default values for CMR:
= 1.0 for unreinforced masonry or earthen structure
= 1.5 for special reinforced concrete moment frame
= 2.0 for others

SSF = spectral shape factor, as defined in FEMA P-695. Default value = 1.15

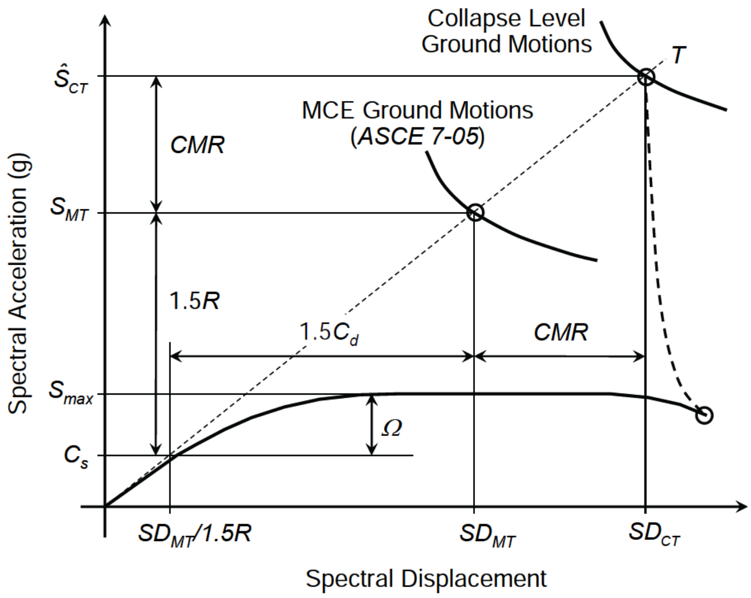


Figure 2. Relating median collapse capacity to the pushover curve (FEMA P-695, Applied Technology Council 2009)

For example, in the U.S., one could calculate C_S from ASCE 7-10 Sec 12.8.1.1, as follows:

$$C_S = \frac{S_{DS}}{\left(\frac{R}{I_e}\right)} \quad (27)$$

4.4.3 Option 2: nonlinear pseudostatic (pushover) analysis

In general, methods based on nonlinear pseudostatic procedures are not advisable for highrise structures. The errors in estimating interstory drift ratios (rather than the global roof drift) rise quickly with increasing height and with the contribution of higher modes. Advanced procedures have appeared in recent works that incorporate multiple modes to improve upon the error of standard single-mode pushover methods. Since structural modelling is typically the most labor-intensive part of the structural analysis, and given that all such improved static procedures are quite complex and unsupported by commercial software, they are not recommended at this point in time. If higher accuracy is sought then, unless the analyst possesses the necessary skills to run such methods, the analyst should resort to option 3, nonlinear dynamic analysis.

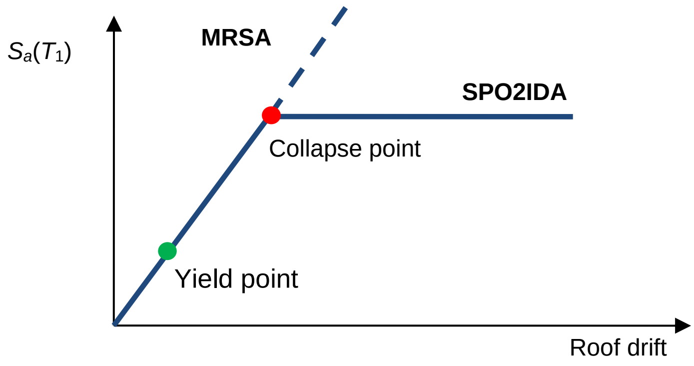


Figure 3. The proposed nonlinear static pushover based method (option 2) as shown in coordinates of the intensity measure versus response (typically used for incremental dynamic analysis). MRSA results, normally applicable only to the elastic region are extended beyond the yield point in accordance with the equal displacement rule for moderate and long period structures. SPO2IDA provides the upper limit of this extension in terms of the intensity measure, showing where collapse is expected to appear (in a median/mean sense). In terms of incremental dynamic analysis, this provides the "flatline" where response becomes (nominally) infinite.

Still, the classic single-mode pushover analysis has been shown to be a viable approach for estimating collapse capacity (Vamvatsikos and Cornell, 2005b), especially if some care is exercised to find the pushover curve that best describes the path followed by the structure to collapse. The present guideline employs this approach for pushover analysis to evaluate global collapse. Elastic modal response spectrum analysis (MRSA) is employed to evaluate structural response given that no collapse has occurred (the portion of the pushover curve in Figure 3 up to the collapse point). In the case of a lowrise building, MRSA should be replaced by a single-mode pushover method, e.g., as described in EN 1998-3 (Comité Européen de Normalisation, 2005) or ASCE 41-06 (American Society of Civil Engineers 2007).

The proposed pushover-based analysis for highrise buildings is as follows:

Step 1: Select a representative hazard spectrum for the analysis. One could employ a uniform hazard spectrum or a design spectrum. These tend to bias the estimate of median response. This guideline recommends using the median spectrum from 7 or more ground motion records, such as the 22 pairs of (unscaled) horizontal components from far field recordings employed by the authors of FEMA P-695 (see Applied Technology Council, 2009, Appendix A). The ground motions are currently available at http://www.csuchico.edu/structural/researchdatabases/ground_motion_sets.shtml and as backup from the GEM Nexus website.

Step 2: Perform MRSA to estimate response values for interstory drifts and peak floor accelerations (see Box 4). Estimate the level of the intensity measure corresponding to the adopted spectrum. The IM can be any of the single period $S_a(T)$'s listed in Section 4.4.1. $S_a(T_{1m})$ is the recommended option, where T_{1m} is the mean T_1 of all index buildings. To estimate the median response for any other level of the IM, linearly scale the results found for the present value of $S_a(T)$.

Step 3: Perform a nonlinear static pushover using a first-mode load pattern (or a load pattern of, e.g., parabolic shape that incorporates higher modes) until the maximum strength is reached. Then continue pushing either with (a) the original pattern or (b) a uniform or (c) an inverse triangular (the original pattern turned upside down). Adopt whichever pushover curve is the most aggressive (lower strength and lesser ductility) of the three. Apply the SPO2IDA tool (https://www.atcouncil.org/projects/ATC-58-1/FEMAP-58-3_Tools.zip) to get the median and dispersion of collapse capacity. For each index building supply the building's own T_1 to SPO2IDA. Then use the spectrum of step 1 to find the ratio of S_a at the period adopted for the IM (T_{1m} being the recommended) over S_a at period T_1. Use this ratio to convert (by simple multiplication) the resulting median $S_a(T_1)$ value of collapse to the period of the common IM. Note that while SPO2IDA also provides comprehensive roof drift information at all levels of intensity, these only correspond to the first mode and are, thus, not considered representative of a tall structure.

Step 4 (optional): If using only three index buildings, proceed with the supplied information: median responses at each IM level and median and logarithmic standard deviation of collapse capacity. If using seven index buildings, then the logarithmic standard deviation of response at each IM level, given that no collapse has happened, is also required. For interstory drifts, assume a uniform 10% initial logarithmic standard deviation for any story drift given the IM level up to the (nominal) yield value of S_a (roughly estimated as the yield strength from the static pushover divided by the weight) and then linearly interpolate between this yield value and the median collapse IM, assigning the logarithmic standard deviation of collapse capacity at the median collapse IM. For peak floor accelerations, similarly interpolate by assigning 1.3 times the logarithmic standard deviation of collapse capacity at yield and a logarithmic standard deviation of 0.3 at collapse. (Peak floor accelerations saturate after yield, thus losing variability.) To generate discrete values similar to the results of nonlinear dynamic analyses from different ground motion records (needed to apply Monte Carlo for uncertainty propagation) that can represent the dispersion of each response EDP given the IM value, simple stratified sampling can be employed. Assuming a lognormal distribution, it is sufficient to get 7 such discrete points for each response type (each peak floor acceleration and story drift) for each level of IM. To do so, evaluate $DP_i(x) = m_{dp}(x) \cdot \exp[\beta_{dp}(x) \cdot K_i]$ where $m_{dp}(x)$ and $\beta_{dp}(x)$ are the median and logarithmic standard deviation of the demand parameter at intensity measure level x, while the standard variates are K_i = -1.4652, -0.7916, -0.3661, 0.0, 0.3661, 0.7916, 1.4652, for i =1,2,…,7.

The proposed pushover-based analysis for low- and midrise buildings is as follows:

Step 1: Same as for highrise buildings

Step 2: Perform a nonlinear static pushover analysis using a first-mode load pattern. Informed analysts may employ alternative nonlinear static procedures at will (e.g., adaptive pushover, different load patterns deemed more suitable to a given structure etc.). For some classes of buildings there are also methods such as DBELA (Calvi 1999, Crowley et al. 2004) and FaMIVE (D'Ayala and Speranza 2003) for directly defining the pushover curve and all pertinent drift responses without performing an actual analysis. Regardless of the method employed, the IM can be any of the single period $S_a(T)$'s listed in Section 4.4.1. $S_a(T_{1m}$,5%) is the recommended option, where T_{1m} is the mean T_1 of all index buildings. Define a range of equally spaced values for the IM of choice, e.g., at steps of 0.1g. Then, use the spectrum of step 1 to find the ratio of S_a at T_1 (i.e. the analysed building's own fundamental period) over S_a at the period adopted for the IM. Use this ratio to convert (by simple multiplication) levels of the common IM to $S_a(T_1$.5%) values for the specific building. Employ either EN 1998-3 (Comité Européen de Normalisation 2005), ASCE 41-06 (American Society of Civil Engineers 2007) or the SPO2IDATool (Vamvatsikos 2010) to transform $S_a(T_1$, 5%) values, into strength ratio R values, then to roof displacements (or equivalent SDOF displacements) and finally to each story's interstory drifts. Estimate corresponding peak floor acceleration values according to Box 4. In the end, the analyst should have the interstory drift and peak floor acceleration values for each story and at each level of the common IM adopted for the set of index buildings.

Step 3: Apply the SPO2IDATool (Vamvatsikos 2010) to get the median and dispersion of collapse capacity. For each index building supply the building's own T_1 to SPO2IDA. Use the ratio of IM over $S_a(T_1)$ (i.e. the inverse of the ratio derived in the previous step for the selected spectrum) to convert the resulting median $S_a(T_1)$ value of collapse to the common IM period (by simple multiplication). If SPO2IDA has also been used in the previous step, then step 2 and step 3 actually become merged.

Step 4 (optional): Same as for highrise buildings.

Box 4: Multi-modal estimation of peak floor accelerations

One must estimate absolute peak floor acceleration (PFA) at each floor of each index building with acceleration-sensitive components. When employing a pushover-based method, estimation of PFAs is sensitive to higher modes, even when their effective mass is relatively low. Thus, it is recommended to use the following simplifying method that uses a square-root-sum-of-squares rule over all N modes (or at least those accounting for a minimum 98% of the total mass) to approximate PFAs at each floor. Then, the elastic PFA demand in story s can be estimated by applying the square-root-sum-of-squares (SRSS) rule to the pseudo acceleration response spectrum.

$$PFA_s \approx \sqrt{PGA^2 c_s^2 + \sum_{j=1}^{N} \left(\Gamma_j \, \phi_{j,s} \, S_a(T_j,\zeta)\right)^2 + 2\, PGA \, c_s \sum_{j=1}^{N} \left(\Gamma_j \phi_{j,s} S_a(T_j,\zeta)\right)}$$

Where c_s rules the spatial contribution of PGA:

$$c_S = 1 - \sum_{j=1}^{N} \left(\Gamma_j \, \phi_{j,s}\right)$$

The above yields PFA_0 = PGA at the ground floor. Γ_j is the modal participation factor for mode j and $\varphi_{j,s}$ is its corresponding component for story s. $S_a(T_j,\zeta)$ is the spectral acceleration at mode period T_j and damping ratio ζ for elastic response. In other words, for any point along the pushover where the strength ratio $R \leq 1$. For higher values of R, i.e. responses past the nominal yield point of the capacity curve, the PFA values at $R = 1$ should be used. To estimate the PGA and S_a values, the elastic response spectrum used for MRSA should be employed. Note that the main assumption in the above equation is that the frequencies are uncorrelated and furthermore the frequencies are uncorrelated with respect to PGA. This is only true for stories well above the ground level. Thus, some unconservative bias reaching up to 30% can be expected, a fact that is reflected in the value of the β_m factor (Eq. 6) for adding uncertainty at the component level. An extended CQC rule according to Taghavi and Miranda (2008) can significantly improve the results at the expense of more complex calculations.

Box 5: General guidance on structural modeling

Typically, a considerable part of the effort in vulnerability estimation lies with creating a structural model of the index buildings. In this respect, it is often advantageous to adopt a reduced MDOF model employing story-level (rather than component-level) characterization of mass, stiffness and strength. One estimates story-level characteristics rather than creating a 2D or 3D component-by-component ssl model. Equally importantly, such models can be analysed within a few seconds using either nonlinear dynamic or static methods. Two such types of models are presented together with the option of a conventional detailed model.

A. Stick models

The basic idea stems from the use of fishbone models (Luco et al. 2003, Nakashima et al. 2002) to represent moment-resisting frame buildings using only a single column-line with rotational restrictions for each floor owing to the presence of beams. This idea is hereby simplified to cover many different structural systems as shown in Figure 4. It comprises N nodes for N stories, each with 3 degrees of freedom (horizontal, vertical, rotational) in 2D space. The nodes are connected by N columns in series and further restrained rotationally by N springs representing the strength and stiffness of beams at each floor. All elements are nonlinear, at the minimum having a simple elastic-perfectly-plastic force-deformation (or moment-rotation) behavior with an ultimate (capping) ductility limit (or a dramatic loss of strength) that is explicitly modeled. Element characteristics can be derived using the aggregate stiffness of the columns, piers, walls. and beams in each story, together with the corresponding yield and ultimate displacements or rotations.

Columns may be modeled using lumped-plasticity or distributed-plasticity force-based beam-column elements. Displacement-based elements are not recommended unless every column is represented by at least four such elements, with the ones closer to the ends being considerably smaller to allow for a reliable localization of deformation. In all cases, for each story-level column the analyst should define the moment-rotation characteristics of the element section. Assuming a capped elastic-plastic model (Figure 5) at the very minimum, together with a lumped-plasticity column representation, each story of a given height is characterized at minimum by the following parameters:

(1) Story column plastic hinge, taken to represent the total stiffness and strength of all the columns in the story. The yield and ultimate rotations need to represent the average values across all story columns.

(2) Beam rotational spring, to represent the total stiffness of N_b beams in double curvature bending, the strength of $2N_b$ beam rotational hinges and the average of the corresponding rotational ductilities (including any slab contributions, if thought to be significant).

(3) The story translational mass, to be applied at each story node

P-Delta effects are taken into account by applying appropriate gravity loads and assigning the proper geometric transformation to columns. For use with perimeter (rather than space) frame systems, the use of a leaning column is not necessary as the entirety of the story mass (and gravity load) is applied at the single story node. Still, this means that the area of the column element, but not its moment of inertia, needs to be increased to represent the total column are of both moment-resisting and gravity framing elements.

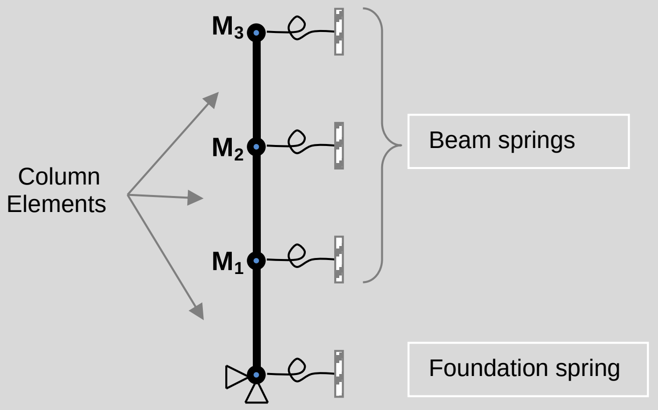


Figure 4. A three-story stick model, showing rotational beam-springs, column elements and floor masses M_1 – M_3

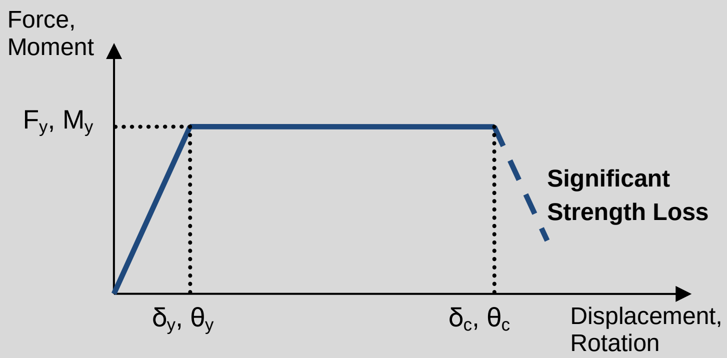


Figure 5. Capped elastic-plastic force-deformation (or moment-rotation) relationship

In general, stick models are not recommended for cases where the building height is larger than three times its width, as the flexural component of deformation due to column elongation may become important. Additionally, they are only appropriate for structures having rigid diaphragms without appreciable plan irregularities. When dual systems are to be modeled, e.g. systems where both structural walls or braces and moment-frames significantly contribute to lateral stiffness and strength, it is advised to employ two stick models side-by-side, connected by horizontal translation constraints to represent the rigid diaphragm.

B. Single-bay frame

Whenever it is desirable to further distinguish the behavior of the column springs to their individual constituents, a single-bay multi-story frame may be employed instead of a simple stick. Each story is now represented by two columns and one connecting beam plus any additional element acting at the story level, such as braces and infill walls. While a complex story-level element would need to be defined for an equivalent stick model, the larger number of elements employed by the single-bay model allows an easier way for defining the behavior of the story. For example, braces and infill walls can be explicitly added by the inclusion of the proper element(s), as for a proper 2D component-by-component model. Definition of beam and column characteristics follows exactly the details laid out for the stick model. The only difference is that all strength and stiffness terms need to be divided equally among the two columns. Similarly, each story's beam has now two distinct plastic hinges. Thus each of those should represent the contribution of N_b beam plastic hinges, where N_b is the number of bays. The same restrictions of use should be observed as for stick-models, although, extended applicability to higher aspect ratios is expected.

C. Detailed 2D/3D models

A proper 2D or 3D model of the full-scale structure needs to be created following either to a great or a lesser detail, making sure that the most important features of the class are retained. The choice of a 2D or 3D model depends on the plan asymmetry characteristics of the building, 3D being most appropriate wherever significant eccentricity (torsion effects) are expected. Appropriate representation of the nonlinear behavior of lateral-load resisting elements and any inherent global or local geometric nonlinearity (e.g., P-Delta effects or brace buckling) is essential. For more details, see Deierlein et al. (2010). It is noted that only engineers well versed in nonlinear structural modeling should follow this approach for vulnerability analysis. It is bound to be quite time-consuming for the average user.

4.4.4 Option 3: nonlinear dynamic structural analysis

Guidance in this section focuses on selecting ground-motion time histories for use in the structural analysis, and then extracting the structural response measures discussed in Section 4.4.1. See Box 5 for general guidance on creating the structural model.

We recommend using incremental dynamic analysis (IDA) for performing the nonlinear dynamic analyses. The analysis produces the conditional distribution of each demand parameter DP conditioned on the intensity measure IM for each of many IM levels, from elasticity to global collapse. The structural model is subjected to nonlinear dynamic structural analysis under a suite of ground-motion accelerograms that are scaled to increasing levels of the IM until collapse is reached. To achieve good assessment resolution, at least 18 ground motion recordings should be employed. Ground motions are available from a number of sources, including FEMA P-695 (far-field set), http://peer.berkeley.edu/products/strong_ground_motion_db.html, and www.cosmos-eq.org. These last two are appropriate for shallow crustal earthquakes in active tectonic regimes anywhere in the world. For a 3D model this should become at least 18 pairs of horizontal components, each pair recorded at the same station and event. For a 2D model this translates to using at least 18 horizontal components, none of which belongs to the same recording. In other words, for 2D analysis only a single component of each pair is used, although it is highly desirable to have both components available to allow the estimation of the geometric mean of spectral acceleration of both components at any given period. If it is considered to be of interest and the model supports its use, the vertical ground motion component may also be added to the corresponding horizontal one(s). As of this writing, the FEMA P-695 far-field ground motions are available at http://www.csuchico.edu/structural/researchdatabases/ground_motion_sets.shtml and as backup from the GEM Nexus website.

For most general cases, the 22 pairs of ground motions comprising the FEMA P-695 far-field ground motion set can be employed as a standard set. As IDA uses only scaling to modify the ground motions, it functions best when far field records are used together with an efficient and sufficient IM. Far-field records contain no traces of directivity (i.e. near source pulses). Typically, for use with vulnerability assessment, instances of soft soil amplification should also be avoided, unless the analyst knows this to be the dominant case. As for the IM, use of $S_{agm}(T_i)$, i.e. the geometric mean of both horizontal components of ground motion at several periods of interest, is considered to be a good choice that significantly reduces the variance and the bias typically associated with large scale factors. Use of a single-period $S_a(T_{1m})$ (also as a geometric mean of both horizontal components) instead should only be considered in cases of strong ground motions and relatively weak buildings where scale factors higher than 3.0 are not expected. Regardless of the choice, contrary to the usual practice when running IDA for a single building, it is important herein to use the same IM for all index buildings to avoid having to do some complex postprocessing later on. See also the relevant discussion in Section 4.4 regarding the selection of ground motions and IMs for additional information.

There are efficient approaches to running IDA, for example the hunt-and-fill algorithm by Vamvatsikos and Cornell (2002) that employs 7-9 runs per record to achieve good results with high computational efficiency. These need some additional postprocessing, though, to derive the necessary distributions of DP conditioned on IM. For analysts familiar with Matlab and OpenSees, a full suite for efficiently running and post-processing IDA results is provided by http://users.ntua.gr/divamva/software.html. A less efficient but simpler algorithm is shown below:

1) Select the constant IM-step, say $\Delta x = 0.1g$ for $T \leq 1sec$, $\Delta x = 0.2g$ for $T > 1sec$ (or as suggested in the discussion of IM choices)
2) Increment IM by the IM step, starting from 0.
3) For each record that has not registered a collapse at an earlier IM, perform the time-history analysis and keep only the following information: (a) whether collapse has occurred, and if not then (b) the peak absolute values of absolute peak floor acceleration (PFA) and peak transient interstory drift (PTD) at each story.

Interpret any of the following conditions as collapse:

(a) The structural analysis does not converge. This is not to be confused with non-convergence caused by ill-prepared models or badly run timehistory analyses. Small time steps and good modelling practices are important.

(b) The peak transient interstory drift ratio on any story exceeds a reasonably high value. For all but very ductile building such as woodframe shearwall buildings, 10% peak transient drift ratio is a reasonably high value to use as a proxy for collapse. For woodframe shearwall construction, 20% peak transient drift can be reasonably assumed to result in collapse. Attention should be paid to make sure that the model has been adequately supplied with details (see minimum modelling requirements discussed earlier) that can help it properly simulate collapse. If ductility limits are not included for all non-linear components of the structure, then this 10% value becomes a completely artificial number to determine collapse for a model that cannot predict it. Its use is acceptable only for realistic models that do show a loss of strength due to P-Delta or in-cycle degradation but simply happen to stay numerically stable up to large drift values.

(c) A non-simulated collapse mode has occurred, e.g. shear failure of a column or joint.

Care should be exercised in cases where records may show non-collapsing response even above the IM level where first collapse was recorded (a phenomenon called structural resurrection). Such results should be ignored and only the point of first collapse should be considered.

At each level of IM, the probabilistic characterization of structural response is achieved by (a) the collapse probability denoted here by $P_c(x)$ equal to the number of records that have reached collapse (N_c) divided by the total number of records and (b) the distribution of responses given x (formally DP|IM) supplied by the non-collapsing records for each demand parameter of interest (peak interstory drift ratios and peak floor accelerations). The latter can be directly represented by the actual DP values registered by each non-collapsing record or by their statistical characterization, e.g. their median and log standard deviation, assuming they are lognormally distributed. In the present work, we recommend the former, as lognormality may or may not be appropriate.

4.5 Step 4: Derive building-level vulnerability functions

Finally, calculate the expected value of damage factor at each value of x as follows, where RCN is the replacement cost (new) of the building, N_h is the number of stories, and f_1 is the fraction of total building replacement cost (new) represented by components in the inventory. For simplified structural analysis,

$$y(x) = P_c(x) + (1 - P_c(x)) \cdot \frac{E\left[C \middle| S = s(x), NC\right]}{RCN} \tag{28}$$

$$E\left[C \middle| S = s(x), NC\right] = \frac{1}{f_1} \cdot \sum_{h=1}^{N_h} \left(E\left[C_a \middle| S_{h,a} = s_{h,a}(x)\right] + E\left[C_d \middle| S_{h,d} = s_{h,d}(x)\right] \right) \ if \ \le 0.6$$
$$= 1.0 \ \ otherwise \tag{29}$$

For nonlinear dynamic structural analysis or user-selected analyses that require multiple structural analyses per intensity measure level,

$$y(x) = P_c(x) + (1 - P_c(x)) \cdot \frac{E\left[C \middle| S = s(x), NC\right]}{RCN} \tag{30}$$

$$E\left[C \middle| S = s(x), NC\right] = \cdot \frac{1}{\left(f_1 \cdot n^*(x)\right)} \cdot \sum_{m=1}^{n^*(x)} \sum_{h=1}^{N} \left(E\left[C_a \middle| S_{h,a} = s_{h,a,m}(x)\right] + E\left[C_d \middle| S_{h,d} = s_{h,d,m}(x)\right] \right) \ if \le 0.6$$
$$= 1.0 \ \ otherwise \tag{31}$$

where $n^*(x)$ is the number of ground motion pairs that did not result in collapse at intensity measure level x, $s_{h,a,m}(x)$ is the geometric mean floor acceleration at floor h in ground motion pair m (excluding cases of collapse), and $s_{h,d,m}(x)$ is the peak transient drift ration at story h in ground motion pair m (excluding cases of collapse). When repair cost exceeds 0.6, the building is commonly considered a total loss, hence the jump to a damage factor of 1.0 when the repair cost exceeds 0.6·RCN.

4.6 Step 5: Mean vulnerability function and uncertainty

4.6.1 Uncertainty

It is usually desirable to estimate uncertainty in the vulnerability function. It is quantified here using the coefficient of variation of loss at each value of x. Let us denote it by $v(x)$. We treat it here as having two distinct contributions:

1. Building-to-building variability. This is the uncertainty in loss arising from the fact that different buildings of the same class will perform differently when subjected to the same level of IM. Different buildings of a given class may vary in terms of number of stories, plan and elevation shape, number of bays, material properties, design strength and stiffness, differences in construction quality, and other features. One can estimate the building-to-building variability using the results of both the 3- and 7-index-building options.
2. Within-building variability. This is the uncertainty in loss at a given level of IM for a particular building, because of (a) record-to-record variability in structural response at a given level of IM, (b) uncertain component capacity, that is, the uncertain level of structural response that will produce prescribed damage states, (c) uncertain unit repair costs, that is, the uncertain cost to repair each component of a given type from a specified damage state, (d) uncertainty in material properties not reflected in the deterministic structural model and (e) other modelling errors, that is, the difference in structural response between how the real building in nature behaves and its how the mathematical idealization estimates its behavior. This guideline document provides procedures for calculating within-building variability only for the 7-index-building option.

4.6.2 One or three index buildings

If the analyst considered one index building, then the mean vulnerability function for the class is given by the vulnerability function for the one index building, from Equation (30).

If the analyst considered three index buildings (poor, typical, and superior), then the mean vulnerability function for the class is given by the simple average of the three, as shown in Equation (32), where $y_k(x)$ for each index building k (k = 1, 2, and 3) is given by Equation (30).

$$y(x)=\frac{1}{3}\sum_{k=1}^{3}y_k(x) \tag{32}$$

If the analyst chose to use a single index building, then we recommend using the function suggested by Porter (2010):

$$v(x)=\frac{0.25}{\sqrt{y(x)}} \tag{33}$$

This curve was regressed from the implied coefficient of variation of damage factor from all HAZUS-MH model building types—old and new, weak and strong, stiff and flexible, brittle and ductile—which jointly exhibit a fairly consistent relationship between *v*(*x*) and *y*(*x*). Note that Equation (33) will tend to overestimate uncertainty as collapse comes to dominate loss.

If the analyst chooses to assess three index buildings, then one can explicitly calculate the between-building, within-class coefficient of variation of damage factor conditioned on mean damage factor because one has three distinct mean vulnerability functions to draw upon. Within-building variability is not explicitly calculated for the three-building approach. To estimate it, one assumes the building-to-building and within-building uncertainties are the equal at a given intensity measure level. Under this assumption, one can calculate the coefficient of variation as follows.

$$v(x)=1.4\frac{\sqrt{\frac{1}{2}\sum_{k=1}^{3}\left(y_k(x)-y(x)\right)^2}}{y(x)} \tag{34}$$

The approximate equivalence of building-to-building and within-building uncertainty is supported by the observed ratio of v(x) for all HAZUS-MH building classes and the analytically derived v(x) from the CUREE-Caltech Woodframe Project (Porter et al. 2002). See Appendix D.2 for details.

Once one has calculated *y*(*x*) and *v*(*x*), the conditional distribution of loss can be taken as lognormal with mean and coefficient of variation as described above (truncating at 1.0), or as beta with bounds 0 and 1 and the same mean and coefficient of variation.

4.6.3 Seven index buildings

If the analyst considered seven index buildings, then one has the data necessary to explicitly calculate mean vulnerability, within-building variability, and between-building variability. In appendix B.2, we provide guidance on Monte Carlo simulation of repair costs. Each simulation produces a simulated value of the damage factor, denoted here by $Y_{k,q,sim}(x)$, where *Y* denotes uncertain damage factor, *k* denotes the index building number, *q* denotes an index to quality level (*q* = 1 denotes generally poor quality components, *q* = 2 denotes a typical mix, and *q* = 3 denotes a generally superior or rugged mix of components), *sim* denotes an index to the simulation number, and *x* denotes the intensity measure level.

To be clear about quality levels: we refer here to 7 index buildings, and each index building is represented by a single structural model. But each index building is provided with three variants: poor, typical, and superior, for a total of 21 index buildings and variants. The structural analysis is performed at the index-building level, and assumes that component quality does not affect structural response. The damage and loss analysis takes place at the variant level, after the structural anlaysis, and where component quality does affect loss. $Y_{k,q,sim}(x)$ is calculated as follows

$$\begin{aligned} Y^*_{k,q,sim}(x) &= \frac{1}{f_1\cdot RCN}\left(\sum_{h=0}^{N_h}\sum_{a=1}^{N_a}\sum_{d=1}^{N_{a,d}}n_{h,a,d}\cdot u_{a,d}+\sum_{h=1}^{N_h}\sum_{r=1}^{N_r}\sum_{d=1}^{N_{r,d}}n_{h,r,d}\cdot u_{r,d}\right) && \text{if not collapsed in sim } k \\ &=1 && \text{if collapsed in sim } k \end{aligned} \tag{35}$$

$$
\begin{aligned}
Y_{k,q,sim}\left(x\right) &= Y^*_{k,q,sim}\left(x\right) \text{ if } Y^*_{k,q,sim}\left(x\right) < Y_{CTL} \\
&= 1 \qquad\qquad \text{if } Y^*_{k,q,sim}\left(x\right) \geq Y_{CTL}
\end{aligned}
\tag{36}
$$

where

RCN = replacement cost new. See Appendix B.2 for notes on simulating *RCN.*

f_1 = inventory construction cost as fraction of RCN

h = index to floor level (in the first sum) or story level (in the second sum). “Floor” refers to the ground level, a floor level, or roof level, where h = 0 refers to the ground floor. “Story” refers to the space between two floors. where *h* =1 refers to the ground story.

N_h = number of stories

a = an index to acceleration-sensitive components

r = an index to drift-sensitive components

N_a = number of acceleration-sensitive component types in the inventory

N_r = number of drift-sensitive component types in the inventory

d = index to damage state

$N_{a,d}$ = number of possible damage states of acceleration-sensitive component type *a*

$N_{r,d}$ = number of possible damage states of drift-sensitive component type *r*

$n_{h,a,d}$ = number of acceleration-sensitive components on floor h of type *a* in damage state *d.* See Appendix B.2 for notes on simulating $n_{h,a,d}$.

$n_{h,r,d}$ = number of drift-sensitive components on floor h of type *r* in damage state *d.* See Appendix B.2 for notes on simulating $n_{h,r,d}$.

$u_{a,d}$ = unit cost to repair acceleration-sensitive components of type *a* from damage state *d.* See Appendix B.2 for notes on simulating $u_{a,d}$.

$u_{r,d}$ = unit cost to repair drift-sensitive components of type *r* from damage state *d.* See Appendix B.2 for notes on simulating $u_{r,d}$.

Y_{CTL} = uncertain damage factor associated with constructive total loss. This is the damage factor above which the building is treated as a total loss. Absent better information, assume Y_{CTL} is normally distributed with mean 0.6 and coefficient of variation 0.1.

Note that Equation (35) gives the damage factor in simulation *k* considering the possibility of non-collapse or collapse, while Equation (36) adjusts this amount to account for the fact that buildings are sometimes treated as a total loss when the repair cost exceeds a threshold quantity referred to as constructive total loss, even if that quantity is less than the replacement cost new of the building.

The mean and standard deviation of the vulnerability function at excitation *x* for index building *k* is given by

$$
y_k\left(x\right) = \frac{1}{3 \cdot N_{sim}} \sum_{q=1}^{3} \sum_{sim=1}^{N_{sim}} Y_{k,q,sim}\left(x\right) \tag{37}
$$

$$
\sigma_k\left(x\right) = \sqrt{\frac{1}{3 \cdot \left(N_{sim} - 1\right)} \sum_{q=1}^{3} \sum_{sim=1}^{N_{sim}} \left(Y_{k,q,sim}\left(x\right) - y_k\left(x\right)\right)^2} \tag{38}
$$

The mean and coefficient of variation of the vulnerability function at excitation x for the asset class is given by

$$y(x)=\sum_{k=1}^{7} w_k \cdot y_k(x) \tag{39}$$

$$v(x)=\frac{1}{y(x)}\sqrt{\sum_{k=1}^{7} w_k \cdot \left(\sigma_k^2(x)+\left(y_k(x)-y(x)\right)^2\right)} \tag{40}$$

where

k = index to index buildings $k \in \{1, 2, \ldots 7\}$

$y_k(x)$ = mean damage factor for index building k at excitation x

$y(x)$ = mean damage factor for the asset class at excitation x

$\sigma_k(x)$ = standard deviation of damage factor for index building k at excitation x

$v(x)$ = coefficient of variation of damage factor for the asset class at excitation x

N_{Sim} = number of simulations of structural response, component damage, and repair cost per index building per level of excitation x; we recommend N_{sim} between 20 and 100.

w_k = moment-matching weight of index building k,

sim = index to Monte Carlo simulations, $sim \in \{1, 2, \ldots N_{sim}\}$

$Y_{k,sim}(x)$ = damage factor for index building k in simulation sim at excitation x.

With $y(x)$, $v(x)$, and an assumed parametric distribution of loss conditioned on x (typically either lognormal or beta bounded by zero and 1), one can estimate the probability of exceeding any specified loss or the loss associated with any specified probability. If desired, higher central moments of vulnerability can be estimated as follows:

$$E\left[Y^n\right]=\sum_{k=1}^{7}\frac{w_k}{N_{sim}}\cdot\sum_{sim=1}^{N_{sim}}\left(Y_{k,sim}(x)-y(x)\right)^n \tag{40}$$

where Y denotes uncertain damage factor for a specimen of the class of interest, and n is any moment of interest $n \in \{2, 3, \ldots\}$. For example, n = 3 refers to skewness of the distribution of damage factor conditioned on a value of IM and n = 4 refers to kurtosis.

5 Illustrative Examples

5.1 Example with one index building: lowrise reinforced concrete shearwall office building in a moderate-seismicity region

5.1.1 Asset definition

Table 6. Index building definition

Asset class (e.g., material, LLRS, height category, occupancy)	
Structural material (if used, from GEM taxonomy)	Concrete, reinforced (CR)
Lateral load resisting system (if used, from GEM taxonomy)	Wall (LWAL)
Broad category (choose one)	Shearwall
Height category (if used, from GEM taxonomy)	1-3 stories (H:1,3)
Occupancy (if used, from GEM taxonomy)	Offices (COM3)
Other attribute 1	
Other attribute 2	
Other attribute 3	
Index building quality (if using 3 index buildings; choose one)	Typical
Index building name (if a particular real building is selected)	CU Business School
Index building model (if using local per-square-meter cost manual)	M.120 in RS Means (2009)
Stories	2
Story height (m)	4
Building height (m)	8
Design year	2009
Construction year	2009
Labor cost as a fraction of total labor + material in construction cost	0.5
Local labor cost as a fraction of US labor cost	1.0
Small amplitude fundamental period of vibration T, sec	0.2
Gamma (default = 1.3)	1.3
Median collapse capacity, $\hat{S}_{CT}$, g, IM = Sa(T, 5%), geomean	0.83
Logarithmic standard deviation of collapse capacity (default 0.8)	0.8
Design base shear as fraction of building weight C_S	0.048
Cost manual reference (if used)	RS Means (2009)
Total building cost (currency per m^2)	$1,364
Total building floor area (m^2)	4,460
Total building construction cost (currency, RCN)	$6.08M million
Fraction f_1 construction cost as fraction of RCN	0.52

Table 7. Ranking of components in decreasing order of contribution to construction cost

	Nonstructural components						Structural
Rank	1	2	3	4	5	6	1
Description	Terminal & package units	Plumbing fixtures	Lighting & branch wiring	Partitions	Interior Doors	Exterior windows (curtain walls)	RC shearwalls
NISTIR 6389 class ID	D3050	D2010	D5020	C1010	C1020	B2020	B1040
FEMA P-58 class ID	D3052.011d	(rugged)	C3034.001	C1011.001d	C1020.001	B2022.035	B1044.043
Demand param (choose one)	PFA	PFA	PTD	PTD	PTD	PTD	PTD
Cost per m^2	$196	$143	$126	$74	$47	$42	$86
Total cost/m^2 these items	$715						
Total cost/m^2	$1,364						
f_1 (frac RCN in inventory)	0.52						

Table 8a. Fragility functions and unit repair costs, nonstructural component rank 1

Component Specification, Rank #1				
NISTIR Class	D3050	Comp. name	Terminal & package units	
FEMA P-58 Class	D3052.011d	Unit	Ea	
Demand param	PFA	Ref (default Pact 1.0)	Pact 1.0	
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation*
1	0.25	0.4	38031	1.05
2				
3				
4				

* This is a large log std dev because PACT uses MECE damage states. Parameters of repair cost are therefore calculated from PACT 1.0 using Equations (1) and (2)

Table 4b. Fragility functions and unit repair costs, nonstructural component rank 2

Component Specification, Rank #2				
NISTIR Class	D2010	Comp. name	Plumbing fixtures	
FEMA P-58 Class	N/A – rugged	Unit	N/A—rugged	
Demand param	N/A – rugged	Ref (default Pact 1.0)	N/A—rugged	
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1				
2				
3				
4				

Table 4c. Fragility functions and unit repair costs, nonstructural component rank 3

Component Specification, Rank #3				
NISTIR Class	D5020	Comp. name	Lighting & branch wiring	
FEMA P-58 Class	C3034.001	Unit	Ea	
Demand param	PFA	Ref (default Pact 1.0)	Pact 1.0	
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1	0.6	0.4	483	0.637
2				
3				
4				

Table 4d. Fragility functions and unit repair costs, nonstructural component rank 4

Component Specification, Rank #4				
NISTIR Class	C1010	Comp. name	Partitions	
FEMA P-58 Class	C1011.001d	Unit	100 lf = 30m	
Demand param	PTD	Ref (default Pact 1.0)	PACT 1.0	
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1	0.0035	0.7		
2	0.0093	0.45		
3				
4				

Table 4e. Fragility functions and unit repair costs, nonstructural component rank 5

Component Specification, Rank #5				
NISTIR Class	C1020	Comp. name	Interior doors	
FEMA P-58 Class	C1021.001	Unit	Each	
Demand param	PTD	Ref (default Pact 1.0)	Porter judgment	
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1	0.024	0.4	500	0.4
2				
3				
4				

(1) PACT does not provide a fragility function for interior doors. The fragility function used here is based on the Porter's judgment. Rationale is as follows: the damage state is "door jams; replace." The door is actually sensitive to peak residual drift. It is assumed that PRD = 0.25·(PTD – 0.005), i.e., peak residual drift is 0.25 times peak transient drift in excess of yield, and that yield occurs at 0.5% drift. These figures are generally taken from unpublished work by Deierlein et al. for FEMA P-58, who estimated PRD as a function of PTD for 4 building types and 4 levels of PTD. It is also assumed that a single damage state occurs: door is jammed, and that door jamming occurs at PRD $\theta = 0.0047$ (based on 3/8 inch residual drift over a door height of 80 inches), with $\beta = 0.4$. For convenience, θ_{PTD} is taken as 0.024. Thus, $\theta_{PRD} = 0.25 \cdot (0.024 - 0.005) = 0.0047$, as desired.

Table 4f. Fragility functions and unit repair costs, nonstructural component rank 6

Component Specification, Rank #6 (typical quality if 7 index buildings)				
NISTIR Class	B2020	Comp. name	Exterior windows (curtain walls)	
FEMA P-58 Class	B2022.035	Unit	Each (4' by 8' panel)	
Demand param	PTD	Ref (default Pact 1.0)	PACT 1.0	
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1	0.0181	0.25	2500	0.17
2				
3				
4				

Table 4g. Fragility functions and unit repair costs, structural component rank 1

Component Specification, Structural Rank #1 (typical quality if 7 index buildings)				
NISTIR Class	B1040	Comp. name	Reinforced Concrete Structural Elements	
FEMA P-58 Class	B1044.043	Unit	Each (900 sf)	
Demand param	PTD	Ref (default Pact 1.0)	PACT 1.0	
Fragility function			Repair cost by damage state	
Damage state	Median capacity	Log std deviation	P_{50} (median cost)	Log std deviation
1	0.0076	0.35	89,254	0.08
2	0.0134	0.45	151,740	0.15
3				
4				

Table 9. Component inventory by story

Rank:	1	2	3	4	5	6	7
Comp. name	Terminal & package units	Plumbing fixtures	Lighting & branch wiring	Partitions	Interior doors	Exterior windows	Shearwalls
Unit	Ea	Ea	Ea	30m	Ea	Ea (4' x 8')	Ea (84 m^2)
Story	Quantity (total)						
1	2	70	60	5	30	80	12.2
2	0	70	60	5	30	80	12.2
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							

5.1.2 Story-level vulnerability function, without collapse

Consider item 1, terminal and package units (2 packaged HVAC units at ground level), subjected to floor-level acceleration x = 0.2g.

$N_D = 1$

$\theta_1 = 0.25$

$\beta_1 = 0.4$

$P_{50} = 37600$

$b = 0.17$

$\beta_m = 0.3$ (pushover analysis)

$$\begin{aligned}
\beta'_{1,1} &= \sqrt{\beta_{1,1}^2 + \beta_m^2} = \sqrt{0.4^2 + 0.3^2} \\
&= 0.5 \\
\theta'_{i,d} &= \theta_{i,d} \cdot \exp\left(1.28 \cdot \left(\beta'_{i,d} - \beta_{i,d}\right)\right) \\
&= 0.25 \cdot \exp\left(1.28 \cdot \left(0.5 - 0.4\right)\right) \\
&= 0.28
\end{aligned} \tag{41}$$

$$\begin{aligned}
p_{1,1}(0.2) &= \Phi\left(\frac{\ln\left(s/\theta'_{1,1}\right)}{\beta'_{i,d}}\right) \\
&= \Phi\left(\frac{\ln\left(0.2/0.28\right)}{0.5}\right) \\
&= 0.25 \\
m_{i,d} &= P_{50,i,d} \exp\left(0.5 \cdot b^2\right) \\
&= \$37600 \cdot \exp\left(0.5 \cdot 1.4^2\right) \\
&= \$100,184
\end{aligned}$$

$$\begin{aligned}
E\left[C_1 \middle| S_{1,1} = 0.2\right] &= N_i \cdot \sum_{d=1}^{N_D} p_{i,d}(s) \cdot m_{i,d} \\
&= 2 \cdot \left(0.25 \cdot 100,184\right) \\
&= \$50,092
\end{aligned}$$

Without showing the work related to the other acceleration-sensitive component, lighting and branch wiring, we evaluated the acceleration-sensitive vulnerability at $S_1 = 0.2g$,

$$\begin{aligned}
E\left[C_a \middle| S_1 = 0.2g\right] &= \sum_{i=1}^{n} E\left[C_i \middle| S_1 = 0.2g\right] \\
&= \$50,092 + \$265 \\
&= \$50,357
\end{aligned}$$

Illustrating for drift-sensitive components at drift = 0.001, and without showing the calculations that lead up to this,

$$
\begin{aligned}
E\left[C_d \,|S_1 = 0.001\right] &= \sum_{i=1}^{n} E\left[C_i \,|S_1 = 0.001\right] \\
&= \$1020 + 0 + 0 \\
&= \$1020
\end{aligned}
$$

5.1.3 Building-level vulnerability

Period T:

T = $0.0488 \cdot 8^{0.75}$
= 0.22 sec

x = Sa(0.3 sec, 5%), g, geomean

Consider Sa(0.3, 5%) = 0.4g

PGA = 0.4 · Sa(0.3g, 5%) = 0.16g

Consider PFA at floor 1, z = 0:

$$
\begin{aligned}
s_{1a}\left(0.4g\right) &= PGA + \varphi\left(h\right) \cdot \left(\Gamma \cdot S_a\left(T, 5\%\right) - PGA\right) \\
&= 0.16g + 0 \cdot \left(1.3 \cdot 0.4g - 0.16g\right) \\
&= 0.16g \\
&\leq S_{\max}
\end{aligned}
$$

At upper floors we need

$$
\varphi\left(h\right) = \frac{z^2}{Z^5} \cdot \frac{\left(70Z^3 - 40Z^2 z + 5Zz^2 + 2z^3\right)}{37} \qquad \text{shearwall building}
$$

Evaluating at z = 0, 3, and 6m,

Floor	Height, m	Mode shape
1	0	0.000
2	4	0.348
R	8	1.000

Now consider collapse fragility:

S_S = 0.30g (near Boulder, CO)

Fa = 1.2 (site class C)

S_{MS} = 1.2·0.30g = 0.36g

S_{DS} = 2/3·S_{MS} = 0.24g

R = 4

I = 1

C_S = S_{DS}/(R/I) = 0.06g

CMR = 2.0 (RCSW building)

SSF = 1.15

$$
\begin{aligned}
\hat{S}_{CT} &= C_S \cdot 1.5 \cdot R \cdot CMR \cdot SSF \\
&= 0.06 \cdot 1.5 \cdot 4 \cdot 2.0 \cdot 1.15 \\
&= 0.828g
\end{aligned}
$$

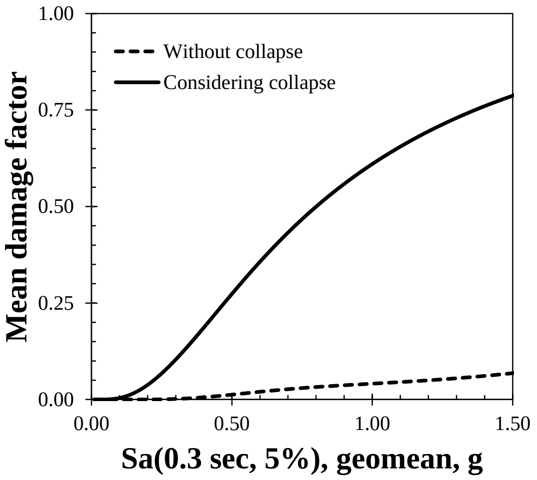
At S_a(0.3 sec, 5%) = 0.4g,

$$
\begin{aligned}
P_c(0.4g) &= \Phi\left(\frac{\ln\left(x/\hat{S}_{CT}\right)}{\beta_{TOT}}\right) \\
&= \Phi\left(\frac{\ln(0.4/0.828)}{0.8}\right) \\
&= 0.18
\end{aligned}
$$

$$
\begin{aligned}
y(0.4g) &= P_c(0.4g) + \left(1 - P_c(0.4g)\right)\cdot\left(\frac{\frac{1}{f_1}\sum_{h=1}^{N} E\left[C\middle|S_h = s_h(0.4g)\right]}{RCN}\right) \\
&= 0.18 + (1-0.18)\cdot\left(\frac{\frac{1}{0.52}(17,430+317)}{6,080,000}\right) \\
&= 0.19
\end{aligned}
$$

$$
\begin{aligned}
v(0.4g) &= \frac{0.25}{\sqrt{y(0.4g)}} \\
&= \frac{0.25}{\sqrt{0.19}} \\
&= 0.58
\end{aligned}
$$

Repeating these calculations at x ∈ {0.01, 0.02, … 3.0g} results in the vulnerability functions shown in Figure 6. Note that collapse governs vulnerability in this case, owing to fairly low design capacity.



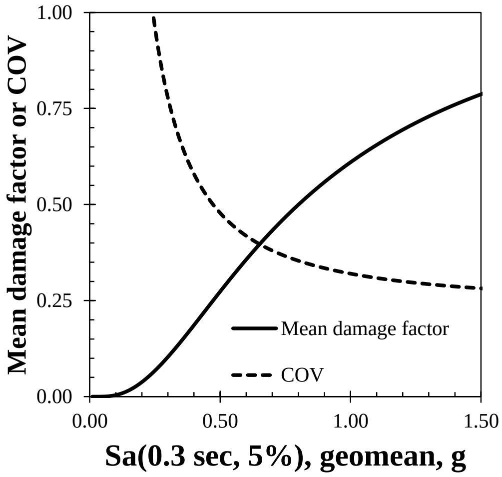


Figure 6. Seismic vulnerability of 1-3 story RCSW office building in region of 0.17 ⩽ S_{MS} <0.5g, 1 index building example

5.2 Example with three index buildings

The approach is the same as in Example 1, but with a poor and superior quality variant. For illustration purposes this example merely substitutes highly fragile, moderately fragile, and low-fragility FEMA P-58 equivalents for the NISTIR component categories, and increases or decreases median collapse capacity by a factor of 1.3 for superior or poor quality, respectively. Figure 7 shows the resulting vulnerability functions. Note that in this particular case, the typical case (same as in the previous example) is very similar to the mean, though we do not yet know how generally true this is. The coefficient of variation has the same trend as in the 1-index-building case but is somewhat higher, which shows that a supposedly more sophisticated model does not necessarily produce lower uncertainty.

Table 10. FEMA P-58 component types used for poor, typical, and superior quality variants of lowrise RC shearwall office building

Component Description	Poor	Typical	Superior
Exterior Windows (Curtain Wall)	B2022.032	B2022.035	B2022.071
Partitions	C1011.001a	C1011.001c	C1011.001b
Interior Doors	C1021.001	C1021.001	C1021.001
Plumbing Fixtures (rugged)			
Terminal & Package Units	D3052.011b	D3052.011d	D3052.013k
Lighting & Branch Wiring	C3034.001	C3034.001	C3034.002
RC shearwalls	B1044.073	B1044.013	B1044.041
Other attributes that vary between index buildings			
	Poor	Typical	Superior
Number of stories	3	2	1
S_S, g	0.2	0.3	0.4

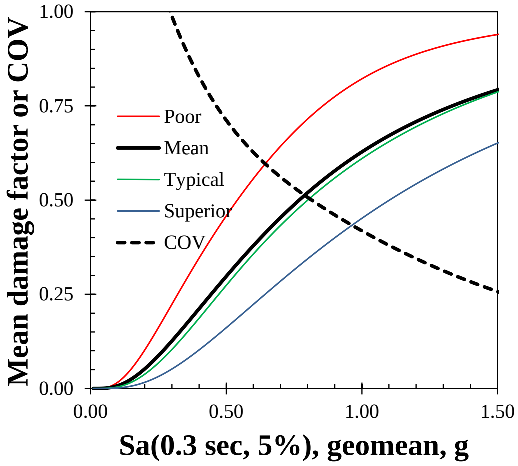


Figure 7. Seismic vulnerability of 1-3 story RCSW office building in region of 0.167 ⩽ S_{MS} <0.5g, 3 index buildings example

5.3 Example with seven index buildings

For an example presenting the derivation of vulnerability functions for a class of highrise office buildings whose lateral force resisting system is a reinforced concrete moment resisting frame, and which is designed according to the International Building Code after 2000 and located in seismic design category D (S_{D1} ≥ 0.2 g), USA, see the companion document by Kazantzi et al. (2013). Therein, seven index buildings selected by moment-matching are analysed in detail using nonlinear dynamic analysis and detailed 2D models.

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APPENDIX A Cho, I. and Porter, K. (2013) Universal Methodology to Construct Building Classes using Multi-Dimensional Moment Matching

APPENDIX B Seven-building asset definition tables; notes on simulation

B.1 Notes on number of stories, degree of vertical irregularity, and degree of vertical irregularity

For buildings on hillsides where the lowest story has varying heights above grade, above-grade stories include all those where at least half the story along at least one façade is above grade.

Degree of vertical irregularity is defined here as the maximum ratio of the height of story h to the height of story h + 1. Typically this is the ratio of the height of the ground story to the height of the story above it.

Degree of plan irregularity is defined here as follows:

- For L-shaped and T-shaped buildings: the smaller ratio of the plan width on one side of a plan setback (the wider wing) to the plan width on the other side of the setback (the narrower wing). By "smaller" is meant that the ratio is calculated in each of two orthogonal directions for each plan setback. The smaller of the two ratios is the degree of plan irregularity.
- For C, E, Z and other shapes: the larger ratio of the plan width on one side of a plan setback (the wider wing) to the plan width on the other side of the setback (the narrower wing). By "larger" is meant that the ratio is calculated in each of two orthogonal directions for each plan setback. The larger of the two ratios is the degree of plan irregularity.

B.2 Notes on Monte Carlo simulation

The analyst who performs Monte Carlo simulation and uses 7 index buildings is assumed here to possess sufficient skills so as not to require a detailed explanation of how to perform the simulation. The following notes are provided to clarify some issues that will arise during the simulation process.

Uncertainty in the asset definition. The 7 index buildings are deemed to capture the bulk of the uncertainty in the asset definition, i.e., the variability between buildings in an asset class. In addition, the within-building variability is deemed to be captured by the uncertain replacement cost new (RCN), the probabilistic depiction of the detailed (FEMA P-58) component categories, together with uncertainty in the hazard analysis, damage analysis, and loss analysis, as described next. By "probabilistic depiction of the detailed (FEMA P-58) component categories" we refer to the fact that each NISTIR component category can be associated with up to 3 FEMA P-58 categories, generically labelled in the inventory tables as poor, typical, and superior quality, each with an associated probability of usage. Absent better information, take *RCN* as normally distributed with a coefficient of variation equal to 0.1.

Uncertainty in the hazard analysis. When performing a structural analysis to determine response conditioned on ground motion, do not include uncertainty in IM given magnitude, distance, and other source, path, and site uncertainties. If the structural model is nonlinear pseudostatic, include uncertain structural response conditioned on IM. If the structural analysis is nonlinear dynamic, for each simulation, randomly select among the ground-motion time histories.

Uncertainty in the structural analysis. If the structural model is a nonlinear dynamic model, it is reasonable to assume that uncertainty in the ground-motion time history conditioned on IM and the uncertainty in the damage analysis swamp uncertainty in structural response given the ground-motion time history. While the analyst is free to include uncertainty in the structural model, this guideline does not require it. Those uncertainties can include member dimensions, material strength, elastic modulus, and elastic damping ratio.

Uncertainty in the damage analysis. The probability mass function of component damage state conditioned on structural response is as prescribed by Equation (3). We recommend simulating damage state for all components of a given type on a given story (for drift-sensitive components) or given floor (acceleration-sensitive) as if their damage were perfectly correlated, that is, they all share the same damage state. Different components of different types on the same story or floor can be treated as independent, conditioned on structural response. Components on different stories or floors can also be treated as independent, conditioned on structural response.

Uncertainty in the loss analysis. The unit cost to repair a component from damage state *d* is deemed to be uncertain, and PACT suggests that some are normally distributed and some are lognormal. The coefficients of variation are often so small that there is no practical difference between the two. For simplicity it is assumed here that all unit costs are lognormally distributed, although the analyst is free to apply the probability distribution indicated in the PACT database. Their 10th, 50th, and 90th percentiles are denoted by p10, p50, and p90 in the PACT database. For each simulation, simulate the unit repair cost (the cost to repair a single component from a particular damage state) for each component type and damage state once and applying the same unit cost to all components of the same type and same damage state.

Correlation. Damage among similar components has an unknown degree of correlation. Until evidence to the contrary arises, the analyst can assume for the sake of simplicity that damage among specimens of the same component category on the same floor is perfectly correlated and all other correlations are zero. Unit repair costs among similar components has an unknown degree of correlation. Until evidence to the contrary arises, the analyst can assume for the sake of simplicity that unit repair costs among specimens of the same component category are perfectly correlated and all other correlations are zero.

B.3 Seven-building asset definition tables

Complete the following set of tables, one set for each index building.

Table 11. Index building definition (7 index buildings)

Asset class (e.g., material, LLRS, height category, occupancy)	
Structural material (if used, from GEM taxonomy)	
Lateral load resisting system (if used, from GEM taxonomy)	
Broad category (choose one)	Frame, shearwall, mixed
Height category (if used, from GEM taxonomy)	
Occupancy (if used, from GEM taxonomy)	
Other attribute 1	
Other attribute 2	
Other attribute 3	
Index building ID (choose one)	0, 1, 2, 3, 4, 5, 6
Index building weight (weights for IDs 0-6 sum to 1.0)	
Moments of top uncertainties	
Number of stories (mean and standard deviation)	Mean: Standard deviation:
Vertical irregularity (mean and standard deviation)	Mean: Standard deviation:
Plan irregularity (mean and standard deviation)	Mean: Standard deviation:
Index building attributes	
Number of stories	
Vertical irregularity (typically story 1 height/story 2 height)	
Plan irregularity	
Index building name (if a particular real building is selected)	
Index building model (if using local per-square-meter cost manual)	
Story height (m)	
Building height (m)	
Design year	
Construction year	
Labor cost as a fraction of total labor + material in construction cost	
Local labor cost as a fraction of US labor cost	
Small amplitude fundamental period of vibration T, sec	
Gamma (default = 1.3)	
Median collapse capacity, $\hat{S}_{CT}$, g, IM = Sa(T, 5%), geomean	
Logarithmic standard deviation of collapse capacity (default 0.8)	
Design base shear as fraction of building weight C_S	
Cost manual reference (if used)	
Total building cost (currency per m^2)	
Total building floor area (m^2)	
Total building construction cost (currency)	
Fraction f_1 inventory total construction cost as fraction of RCN	

Table 12. Ranking of components in decreasing order of contribution to construction cost (7 index buildings)

	Nonstructural components						Structural	
Rank	1	2	3	4	5	6	1	2
Description								
NISTIR 6389 class ID								
Demand parameter (PFA or PTD)								

Table 13a. Fragility functions and unit repair costs, nonstructural component rank 1 (7 index buildings)

Nonstructural Component Specification, Rank #1 (typical quality variant)				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Superior quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Poor quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				

Table 13b. Fragility functions and unit repair costs, nonstructural component rank 2 (7 index buildings)

Nonstructural Component Specification, Rank #2 (typical quality variant)				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Superior quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Poor quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				

Table 13c. Fragility functions and unit repair costs, nonstructural component rank 3 (7 index buildings)

Nonstructural Component Specification, Rank #3 (typical quality variant)				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Superior quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Poor quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				

Table 13d. Fragility functions and unit repair costs, nonstructural component rank 4 (7 index buildings)

Nonstructural Component Specification, Rank #4 (typical quality variant)				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Superior quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Poor quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				

Table 13e. Fragility functions and unit repair costs, nonstructural component rank 5 (7 index buildings)

Nonstructural Component Specification, Rank #5 (typical quality variant)				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Superior quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Poor quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				

Table 13f. Fragility functions and unit repair costs, nonstructural component rank 6 (7 index buildings)

Nonstructural Component Specification, Rank #6 (typical quality variant)				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Superior quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Poor quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				

Table 13g. Fragility functions and unit repair costs, structural component rank 1 (7 index buildings)

Structural Component Specification, Rank #1 (typical quality variant)				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Superior quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Poor quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				

Table 13h. Fragility functions and unit repair costs, structural component rank 2 (7 index buildings)

Structural Component Specification, Rank #2 (typical quality variant)				
NISTIR Class		Comp. name		
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Superior quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				
(Poor quality variant)				
FEMA P-58 Class		Unit		
Demand param	PFA or PTD	Ref (default Pact 1.0)		
Fragility function			Repair cost by damage state	
Damage state	Median	Beta	P_{50} (median)	Log standard deviation (b)
1				
2				
3				
4				

Table 14. Component inventory by story

	Nonstructural components						Structural	
Rank:	1	2	3	4	5	6	1	2
Name								
Unit								
Story	Quantity (total)							
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								
16								

APPENDIX C ASCE 7-10 Table 12.2-1

APPENDIX D Other commentary

D.1 Summary of class partitioning

This technique is an alternative to moment matching for selecting five or more index buildings to represent the variability of features in the population. It constitutes an actual partitioning of the population into a set of collectively exhaustive and mutually exclusive subclasses of buildings each of which is represented by a single index building. Similarly to moment-matching, the overall properties of the building population are simply approximated by the joint probability mass function (PMF) established by the index buildings. In simpler terms, this means that the distribution properties (e.g., mean and standard deviation of) the population are assessed by condensing the population to just the index buildings used, to each of which a certain weight is assigned, according to its actual membership (percentage of buildings it represents) in the entire population. For large statistical samples, formal clustering methods (e.g., k-means clustering) need to be used. For most simple cases, though, the intuition and knowledge of an analyst that is intimately familiar with his or her dataset will be enough to select a number of appropriate index buildings, again as characterized by k significant properties, and the appropriate probabilities of occurrence (or weights), p_i. Conceptually, this approach can be aided by loosely following these two steps:

1) Splitting: For each building characteristic, 2 or 3 subclasses are determined that partition the population to distinct parts, each of which represents at least 15% of the population's distribution for the specified characteristic

2) Merging: The total number of possible partitions for the entire population is between 2^k or 3^k subclasses (actually 2 to the number of dimensions split in two times 3 to the number of dimensions split in three parts). Starting from the smaller subclasses, any adjacent subclasses whose participation to the overall population is found to be less than, say, 5% should be concatenated and the sum of their participation percentage assigned to the new wider subclass. Subclasses with participation larger than 15% should not be merged with others. This process is terminated when the desired number of subclasses is reached, typically 7-12.

D.2 Approximate equivalence of building-to-building and within-building uncertainty

In Porter et al. (2002), building-specific seismic vulnerability functions were calculated for 19 variants of 4 woodframe index buildings, using assembly-based vulnerability (ABV, defined initially in Beck et al. 1999 and more clearly in Porter 2000). ABV is essentially the same as 2nd-generation performance based earthquake engineering as adopted by the Pacific Earthquake Engineering Research (PEER) Center and later in FEMA P-58, and now adapted to classes of building in the present guidelines. Porter et al. (2002) calculated the mean and coefficient of variation of damage factor for each index building at each of 20 levels of 5% damped elastic spectral acceleration response, and in one place plotted COV as a function of MDF. Figure 8 shows that relationship for nine variants of two index buildings, and a regression line for one variant of each index building. As others had previously found (W. Graf, personal communication 2002), and as shown in the figure, COV tends to be inversely related to MDF.

In later work, Porter (2010) showed that one can infer the uncertainty in the HAZUS-MH analytical vulnerability functions for its building classes, along with the mean damage factor. Plotting COV versus MDF for 51 levels of IM and 128 building classes, one sees a similar inverse relationship between COV and MDF, as shown in Figure 9. What is relevant here is that the curves in Figure 8 illustrate within-building variability associated with record-to-record variability, uncertainty in structural model parameters, and uncertainty in component damage and unit repair costs. Figure 9 reflects total variability of the class-level vulnerability function, both within-building and building-to-building variability. The ratio of the curve in

Figure 9 to either curve in Figure 8 sheds light on the contribution of between-building variability to total variability. Figure 10 extrapolates all three curves to MDF = 0.5 and plots the ratio of the within-building COV from the CUREE-Caltech index buildings to the total COV from the HAZUS-MH vulnerability functions. The ratio varies with MDF, but is generally around 1.4, which suggests that building-to-building variability is approximately equal to within-building variability, at least for building classes defined as broadly as those in HAZUS-MH. This is the ratio one would get if one were to estimate total uncertainty as the square root of the sum of the squares of within-building and building-to-building standard deviation of loss, and the two were equal.

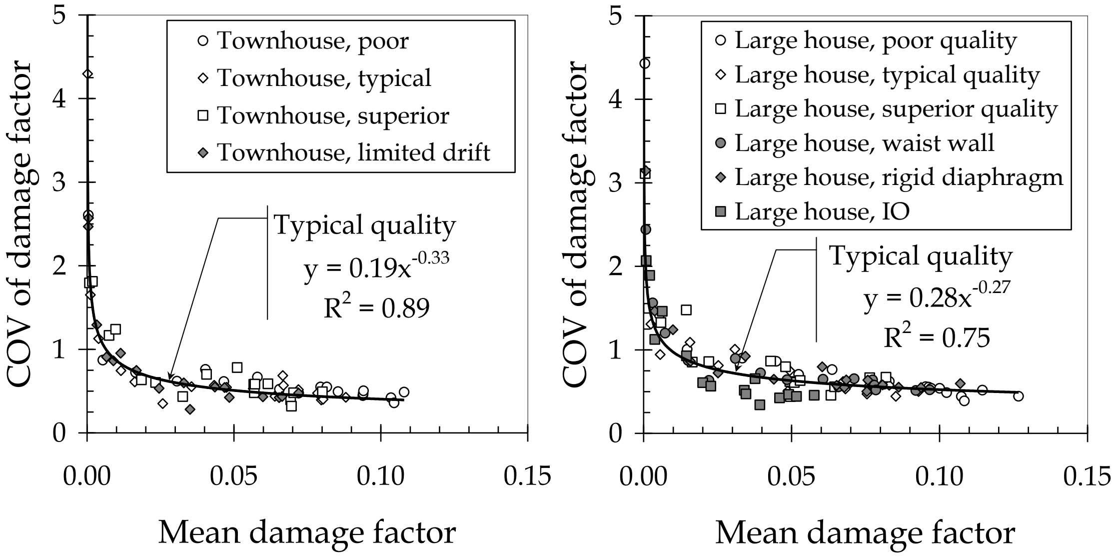

Figure 8. COV versus mean damage factor for 10 variants of 2 index buildings from the CUREE-Caltech Woodframe Project (Porter et al. 2002).

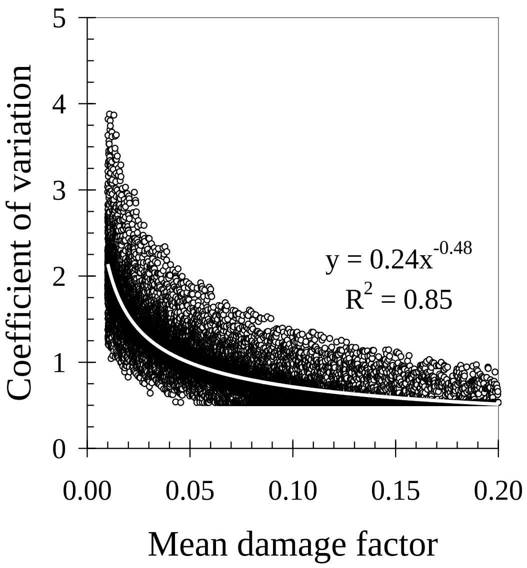

Figure 9. COV versus mean damage factor for 128 HAZUS-MH building classes at each of 51 IM levels (Porter 2010)

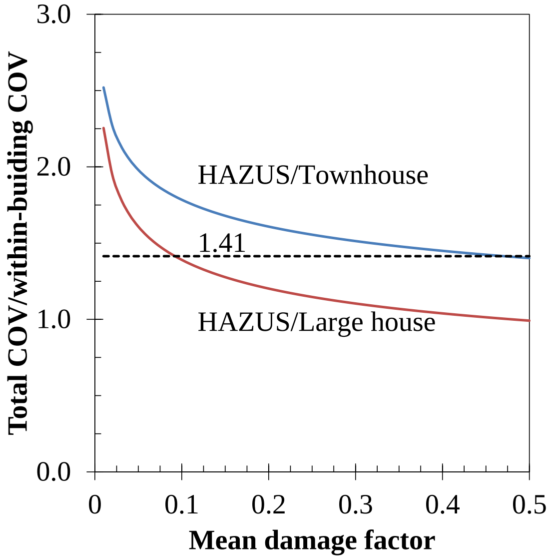

Figure 10. Ratio of total COV to within-building COV as a function of MDF