Simplified Estimation of Economic Seismic Risk for Buildings

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A seismic risk assessment is often performed on behalf of a buyer of commercial buildings in seismically active regions. One outcome of the assessment is that a probable maximum loss (*PML*) is computed. *PML* is of limited use to real-estate investors as it has no place in a standard financial analysis and reflects too long a planning period. We introduce an alternative to *PML* called probable frequent loss (*PFL*), defined as the mean loss resulting from shaking with 10% exceedance probability in 5 years. *PFL* is approximately related to expected annualized loss (*EAL*) through a site economic hazard coefficient (*H*) introduced here. *PFL* and *EAL* offer three advantages over *PML*: (1) meaningful planning period; (2) applicability in financial analysis (making seismic risk a potential market force); and (3) can be estimated using a single linear structural analysis, via a simplified method called linear assembly-based vulnerability (LABV) that is presented in this work. We also present a simple decision-analysis framework for real-estate investments in seismic regions, accounting for risk aversion. We show that market risk overwhelms uncertainty in seismic risk, allowing one to consider only expected consequences in seismic risk. We illustrate using 15 buildings, including a 7-story nonductile reinforced-concrete moment-frame building in Van Nuys, California, and 14 buildings from the CUREE-Caltech Woodframe Project. [DOI: 10.1193/1.1809129]

INTRODUCTION: SEISMIC RISK IN REAL-ESTATE INVESTMENT DECISIONS

Seismic risk enters into several important real-estate decision-making processes: purchase of investment property, performance-based design of new structures, seismic rehabilitation of existing buildings, and decisions regarding the purchase of earthquake insurance, for example. In such situations, it matters who the decision makers are, how they make decisions, what aspects of seismic risk most concern them, how long their planning horizon is, and other parameters. We focus on one of the more common seismic risk decision situations: the purchase of existing commercial property by real-estate investors in seismic regions. (The most common situation is probably purchasing a home in seismically active regions.)

Economic seismic risk to these properties is assessed every time the property changes hands, on the order of every five to ten years. By contrast, a building is designed and built only once. Thus the most common opportunity for market forces to

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bring about seismic-risk mitigation for commercial properties is at times of sale. Anecdotal evidence suggests that these are mostly missed opportunities: risk is typically not mitigated, even in more vulnerable buildings.

This can be partly explained by considering the context in which seismic assessments are performed. During virtually every sale of an existing commercial building, the buyer assesses the building's investment value using a financial analysis that considers revenues and expenses, rent roll, market leasing, physical condition, and other property information. The investor makes his or her bidding decision based on projected income and expenses, using one or more of the economic performance metrics of net present value, net operating income, cashflow, internal rate of return, and capitalization rate.

The input to this financial analysis is typically provided by a real-estate broker representing the seller, whose figures the investor checks and modifies during a due-diligence study. Many of the inputs are known values—number, duration, and income from current leases, for instance—but many are uncertain. Vacancy rates, market rents, and other important parameters fluctuate significantly and unpredictably, leading to substantial uncertainty in the future economic performance of a property. In the face of these uncertainties, the bidder usually estimates investment value using best-estimate inputs and then again with deterministic sensitivity studies to probe conditions that would lead to poor performance (higher future vacancy rates, for example). The future cost to repair earthquake damage is not one of the parameters the bidder uses in the financial analysis. This is important: seismic risk is not a market quantity.

PROBABLE MAXIMUM LOSS: A COMMON METRIC FOR SEISMIC RISK

The real-estate market is not wholly without forces to influence seismic-risk mitigation. The due-diligence study typically includes an engineering assessment of the condition of the property, which itself typically includes an estimate of the earthquake probable maximum loss (*PML*). *PML* is by far the dominant earthquake risk parameter in financial circles.

Interestingly, the earthquake *PML* has no standard quantitative definition, as pointed out by Zadeh (2000). ASTM (1999) grappled with and abandoned an effort to standardize seismic *PML*, producing instead some new terminology. The *PML* nonetheless lingers on. Most working definitions involve the level of loss associated with a large, rare event (Rubin 1991). One definition is that *PML* is the 90th percentile of loss given the occurrence of what building codes until recently called the design basis earthquake, or DBE—an event producing a shaking intensity with 10% exceedance probability in 50 years. Colloquially (and inexactly), this is an upper-bound loss given the 500-year earthquake. More accurately, assuming Poisson arrivals of earthquakes, this shaking level has a mean occurrence rate of $0.00211\ \text{yr}^{-1}$ and a mean recurrence time of 475 years. Because this *PML* is the 90th percentile loss given this level of shaking, the *PML*-level loss can have a much longer mean recurrence time.

Commercial lenders often use *PML* to help decide whether to underwrite a mortgage. It is common, for example, for a commercial lender to refuse to underwrite a mortgage if the *PML* exceeds 20% to 30% of the replacement cost of the building, unless the buyer purchases earthquake insurance—a costly requirement that often causes the investor to decide against bidding. Once the *PML* hurdle is passed, the bidder usually proceeds to ignore seismic risk, for at least three good reasons:

1. **Irrelevant planning period.** Investors plan on the order of five years, making loss corresponding to shaking intensity with a 500-year recurrence time largely irrelevant, too rare even for consideration in a sensitivity study.
2. **Incompatibility with financial analysis.** *PML* is a scenario value, not an ongoing cost that can be reflected in a cashflow analysis.
3. **Custom.** Investors are not required by custom or regulation to include seismic risk in the financial analysis.

Lacking any measure of economic risk beyond *PML*, the bidder has no basis for assessing how the seismic risk of a building should influence the purchase price or for judging whether seismic risk mitigation might be worth exploring. Faced with a high *PML*, the bidder might increase the discount rate used in the financial analysis to reduce the present value of the future net income stream, but no analysis informs the adjustment. This typically closes the matter.

EXPECTED ANNUALIZED LOSS

There is another common term in earthquake loss estimation, namely expected annualized loss (*EAL*) (ASTM 1999), which measures the average yearly amount of loss when one accounts for the frequency and severity of various levels of loss. If one knew *EAL*, one could include it as an operating expense in the financial analysis. Let us consider three ways to estimate *EAL*, from an accurate but information-intensive approach (labeled Method 1 here) to two successively simpler ones (Methods 2 and 3). Method 1 involves evaluation of the seismic vulnerability function for the building and seismic hazard function for the site, and integrating their product to calculate *EAL*. Method 2 takes advantage of the fact that a scenario loss estimate can be shown to be proportional to *EAL*, and uses the constant of proportionality to calculate *EAL* from a scenario loss resulting from an approximately 50-year shaking intensity. Method 3 uses the constant of proportionality as well, and further simplifies the analysis of the 50-year loss using linear spectral analysis. In this work, these three methods are compared using a number of realistic sample facilities, to determine whether the effort involved in seismic risk analysis can be substantially reduced through reasonable simplifications.

EAL METHOD 1: INTEGRATION OF SEISMIC VULNERABILITY AND HAZARD

Assuming independence of intensity and of losses between events, *EAL* can be calculated as

$$EAL = V \int_{S=0}^{\infty} y(s) v(s) ds \tag{1}$$

where V denotes value exposed to loss (e.g., replacement cost of the building), s refers to some seismic intensity measure, $y(s)$ is the mean seismic vulnerability function (defined here as the average level of loss as a fraction of V given the occurrence of s), and $v(s)$ is the average annual frequency of experiencing shaking intensity s. Note that

$$v(s) = \left| \frac{dG(s)}{ds} \right| \tag{2}$$

where $G(s)$ denotes the mean annual frequency of a site experiencing intensity of s or greater, referred to here as the site shaking hazard function. It is convenient to think of shaking intensity in terms of some familiar measure such as 5%-damped elastic spectral acceleration response at a facility's small-amplitude fundamental period, $S_a(T_1)$, but other intensity measures are also valid.

In most practical situations, $y(s)$ and $G(s)$ would be evaluated at a set of $n+1$ discrete intensity values $s_0, s_1, \ldots s_n$. Let us denote these values by $y_0, y_1, \ldots y_n$, and $G_0, G_1, \ldots G_n$, respectively. Let us assume that $G(s)$ varies exponentially between the discrete values of s, i.e.,

$$G(s) = G_{i-1} \exp(m_i(s - s_{i-1})) \quad \text{for} \;\; s_{i-1} < s < s_i \tag{3}$$

where m_i is a negative constant. Equations 2 and 3 then imply for $s_{i-1} < s < s_i$

$$\begin{aligned} v(s) &= -\left.\frac{dG}{ds}\right|_s \\ &= -m_i G(s) \end{aligned} \tag{4}$$

One can estimate m_i as

$$m_i = \frac{\ln(G_i/G_{i-1})}{\Delta s_i} \quad i = 1,2,\ldots n \tag{5}$$

where

$$\Delta s_i = s_i - s_{i-1} \quad i = 1,2,\ldots n \tag{6}$$

Let us assume that the seismic vulnerability function varies linearly between values of s, i.e.,

$$y(s) = y_{i-1} + \frac{\Delta y_i}{\Delta s_i}(s - s_{i-1}) \quad \text{for} \;\; s_{i-1} < s < s_i \tag{7}$$

where

$$\Delta y_i = y_i - y_{i-1} \quad i = 1,2,\ldots n \tag{8}$$

Then EAL is given by

$$
\begin{aligned}
EAL &= V\sum_{i=1}^{n}\left(\int_0^{\Delta s_i}\left(y_{i-1}+\frac{\Delta y_i}{\Delta s_i}\tau\right)(-m_i G_{i-1}\exp(m_i\tau))d\tau\right)+R \\
&= V\sum_{i=1}^{n}\left(-y_{i-1}G_{i-1}\exp(m_i\tau)\Big|_{\tau=0}^{\Delta s_i}-\frac{\Delta y_i}{\Delta s_i}G_{i-1}\left(\exp(m_i\tau)\left(\tau-\frac{1}{m_i}\right)\right)\Big|_{\tau=0}^{\Delta s_i}\right)+R \\
&= V\sum_{i=1}^{n}\left(y_{i-1}G_{i-1}(1-\exp(m_i\Delta s_i))-\frac{\Delta y_i}{\Delta s_i}G_{i-1}\left(\exp(m_i\Delta s_i)\left(\Delta s_i-\frac{1}{m_i}\right)+\frac{1}{m_i}\right)\right)+R
\end{aligned}
\tag{9}
$$

where R is a remainder term for values of $s > s_n$, and has an upper bound of $VG(s_n)$ if $y(s) \leqslant 1$, and where

$$
\tau \equiv s - s_{i-1} \quad s_{i-1} < s < s_i \tag{10}
$$

Let us refer to the method of calculating EAL by Equation 9 as Method 1. It is not easy to perform. To determine $G(s)$ requires an understanding of the local seismic environment: the distance to nearby earthquake faults, the expected rate at which they produce earthquakes of various magnitudes, and the attenuation relationships that give shaking intensity s as a function of magnitude, distance, and other geological parameters. This information is increasingly available. However, to determine $y(s)$ requires either large quantities of empirical post-earthquake survey data (which for various reasons do not exist in reliable form), or laborious engineering damage and loss analyses requiring a skill set beyond that of most engineers, or the exercise of expert opinion, which carries with it the stigma of unverifiability.

Software such as HAZUS (NIBS and FEMA 1999), USQUAKE (see, e.g., EQECAT 1999), and ST-RISK (Risk Engineering 2002) contain pre-evaluated vulnerability and hazard information and can calculate EAL. These programs are widely employed and produce useful information relatively quickly and inexpensively. They treat a wide variety of structure types, and some offer the ability to account for several configurations and other characteristics that affect seismic performance. However, they rely to a significant extent on expert opinion and do not perform structural analysis on a building-specific basis. They are thus insensitive to many of the details that cause performance differences between distinct buildings of the same building type. So what can be done if one wishes to avoid reliance on expert opinion, account for details at the level of standard practice of design, and yet keep the analysis relatively simple? We explore the basis for such a simplified loss-estimation procedure in this paper.

Assembly-Based Vulnerability

A method to assess building vulnerability is essential for loss estimation. A recently developed method called assembly-based vulnerability (ABV) provides a rigorous probabilistic framework for assessing building vulnerability. It considers detailed structural and nonstructural characteristics of the building and accounts for uncertainties in the ground motion, structural features, damageability of structural and nonstructural components, and unit repair costs, to provide a probabilistic description of earthquake response, damage, and loss conditional on shaking intensity. It extends work pioneered by Czarnecki (1973), Kustu et al. (1982), and others that disaggregate a building into categories of components whose damage can be evaluated as a function of the structural response, and whose repair cost can be calculated using construction-cost-estimation techniques. ABV uses a more detailed category system than previous methods to produce vulnerability functions that are more building-specific, among other differences.

Details of ABV have been described elsewhere; see, for example, Beck et al. (1999), Porter (2000), or Porter et al. (2001). Briefly, ABV is a simulation procedure that involves selection of ground-motion time histories, creation of a stochastic structural model, performance of nonlinear time-history structural analyses to determine structural response, assessment of probabilistic damage via component fragility functions, assessment of loss via probabilistic construction cost-estimation, and repetition many times to estimate the probability distribution of loss at various levels of intensity. ABV has proven to be a useful research tool. We have used it to evaluate seismic risk and to perform benefit-cost analysis of seismic-risk mitigation for steel-frame, woodframe, and concrete buildings, and to explore major contributors to the uncertainty in economic seismic risk (Beck et al. 1999, Porter et al. 2002b, Beck et al. 2002, Porter et al. 2002a).

However, ABV is difficult to use in professional practice for estimating $y(s)$ because it requires special skills and software to create the stochastic nonlinear structural model, to perform the hundreds of structural analyses, to model the component damageability, and to calculate repair costs. ABV is not particularly computationally costly. Once set up, the structural analyses for a typical building can be performed overnight in batch mode, and the subsequent damage and loss analyses can be performed in an hour or so. In each of the examples presented here, the structural analyses took on the order of 8 to 12 hours on a common desktop computer. It is the setup that is time-consuming, principally the creation of the structural model.

Some simplifications are possible that can make ABV a more realistic alternative for practitioners to calculate *EAL*, and to produce a probability-based scenario risk measure that is more meaningful to investors than *PML*. The two keys to these simplifications, to be discussed in later sections, are that (1) *EAL* appears to be dominated by nonstructural damage at moderate levels of shaking, where structural behavior is probably well approximated by linear dynamics, and (2) *EAL* can be approximated by the product of a scenario mean-loss level and a site economic hazard coefficient, suggesting that one scenario loss analysis could produce a good estimate of *EAL*.

EAL METHOD 2: USING PROBABLE FREQUENT LOSS

Suppose one estimates the mean loss associated with the shaking intensity that has 10% exceedance probability in 5 years, which corresponds to a return period of approximately 50 years (more accurately, 47.5 years, assuming Poisson arrivals of earthquakes). For convenience, let an earthquake with this intensity be referred to as the economic-basis earthquake, or EBE. Let the mean loss given the EBE be referred to as the *probable frequent loss* (*PFL*), in contrast with the *PML*. There is good reason to define the EBE this way. To test the life safety of a structural design, structural engineers have historically considered upper-bound shaking (10% exceedance probability) during the design life of the building (50 years), referring to this level of shaking as the design-basis earthquake (DBE). If one wants to examine an upper-bound event during the owner's planning period, then it is consistent to use the same exceedance probability (10%) during the owner's planning period (5 years).

Rather than shaking in the EBE, why not use the shaking intensity with 50% exceedance probability in 50 years, a scenario shaking level treated, for example, by *FEMA-356* (ASCE 2000), and which would be only slightly stronger than the EBE? The reason is effective risk communication: EBE is defined for its meaning to the investor, for whom 50 years is too long a planning period and 50% exceedance probability does not bespeak an upper-bound intensity. Our definition of EBE more simply and directly addresses the concerns of the investor.

Let the shaking intensity for the EBE be denoted by S_{EBE}, which again can be measured in terms of S_a or by some other convenient intensity scale. Let us assume there is some intensity level associated with the initiation of damage, and let this intensity be denoted by S_{NZ} (NZ referring to nonzero damage). Let us assume that there is some intensity level, $S_U \geqslant S_{EBE}$, at which the vulnerability function reaches an upper bound, y_U, such that for $s \geqslant S_U$, $y(s)=y_U$. In some cases the upper-bound damage factor might be unity, i.e., one would not pay repair costs in excess of the replacement cost. Let us denote the mean annual frequencies of a site exceeding S_{NZ}, S_{EBE}, and S_U by G_{NZ}, G_{EBE}, and G_U, respectively. We examine a simplified method for computing *EAL* in which we approximate seismic vulnerability and hazard functions by

$$\begin{aligned} y(s) &= 0 && s < S_{NZ} \\ &= a(s-S_{NZ}) && S_{NZ} \leqslant s \leqslant S_U \\ &= y_U && S_U \leqslant s \end{aligned} \tag{11}$$

$$G(s) = G_{NZ} \exp(m(s-S_{NZ})) \tag{12}$$

where a and m are constants. In the case studies presented later, we examine the quality of the approximations in Equations 11 and 12, which reflect a special case of Equation 9 with $n=1$. Here, $s_0=S_{NZ}$, $s_1=S_U$, $G_0=G_{NZ}$, $G_1=G_U$, $y_0=0$, and $y_1=y_U$. Since $y(S_{EBE})=PFL/V$ and $G(S_{EBE})=G_{EBE}$,

$$a = \frac{PFL}{V(S_{EBE}-S_{NZ})} \tag{13}$$

$$m = \frac{-\ln(G_{NZ}/G_{EBE})}{S_{EBE}-S_{NZ}} \tag{14}$$

$$\begin{aligned} S_U &= S_{NZ} + \frac{y_U}{a} \\ &= S_{NZ} + \frac{y_U V(S_{EBE}-S_{NZ})}{PFL} \end{aligned} \tag{15}$$

We can now evaluate EAL. Defining $\tau \equiv s - S_{NZ}$ and $\sigma \equiv s - S_U$, and recalling that $m < 0$,

$$EAL = V\left(\int_{s=S_{NZ}}^{\infty} a(s - S_{NZ})(-mG(s))ds - \int_{s=S_U}^{\infty} (a(s - S_{NZ}) - y_U)(-mG(s))ds\right) \quad (16)$$

$$\begin{aligned}
EAL &= V\left(\int_0^{\infty} (a\tau)(-mG_{NZ}\exp(m\tau))d\tau - \int_0^{\infty} (a\sigma)(-mG_U\exp(m\sigma))d\sigma\right) \\
&= Va\left(\left(-(G_{NZ} - G_U)\left(\exp(m\tau)\left(\tau - \frac{1}{m}\right)\right)\right|_{\tau=0}^{\infty}\right) \\
&= -Va\frac{(G_{NZ} - G_U)}{m} \\
&= \frac{PFL}{(S_{EBE} - S_{NZ})}\frac{(G_{NZ} - G_U)(S_{EBE} - S_{NZ})}{\ln(G_{NZ}/G_{EBE})} \\
&= \frac{(G_{NZ} - G_U)}{\ln(G_{NZ}/G_{EBE})}PFL \quad (17)
\end{aligned}$$

If S_U is significantly greater than S_{NZ}, as expected, then G_U will be small compared with G_{NZ}, which leads to

$$EAL \approx \frac{G_{NZ}}{\ln(G_{NZ}/G_{EBE})}PFL \quad (18)$$

Defining

$$H \equiv \frac{G_{NZ}}{\ln(G_{NZ}/G_{EBE})} \quad (19)$$

leads to the final form:

$$EAL \approx H \cdot PFL \quad (20)$$

where H is referred to as the site economic hazard coefficient. It contains only hazard variables, so it can be mapped as a scalar for a given fundamental period, soil condition, and S_{NZ}. Its units are yr^{-1}. It is a simple matter to calculate the expected present value of future earthquake losses using EAL, given the discount rate (denoted by i), and the planning period (denoted by t):

$$\begin{aligned}
PV &= EAL\frac{(1 - e^{-it})}{i} \\
&= H \cdot PFL\frac{(1 - e^{-it})}{i} \quad (21)
\end{aligned}$$

Equations 20 and 21 still require that one estimate PFL in some way. One choice is to perform an ABV analysis at the intensity level S_{EBE}, including selection of ground-motion time histories, nonlinear time-history structural analysis, damage analysis, loss analysis, and simulation to account for uncertainties in ground motion, mass, damping, force-deformation behavior, component capacity, unit repair cost, and contractor overhead and profit. Let us refer to this approach as Method 2. It is simpler than Method 1 in that it does not require the evaluation of the seismic vulnerability function over the range of all possible shaking levels (as in Method 1). This would reduce the number of analyses by an order of magnitude, but would still require specialized skills and software—one still needs to set up the structural model, for example.

We can further simplify the loss analysis by taking advantage of the fact that at low levels of intensity, around S_{EBE}, the structural response of the facility might be adequately modeled using linear spectral analysis (thus avoiding the time-consuming construction of a nonlinear structural model). Furthermore, only mean loss at S_{EBE} is required, not damage and not an estimate of uncertainty, so we can avoid some aspects of ABV that are intended to quantify damage and uncertainty.

EAL METHOD 3: PFL AND LINEAR ABV

Let us sketch a simplified approach called linear assembly-based vulnerability (LABV), and show how it can be used to calculate *PFL* and *EAL*. It has four steps:

1. *Facility definition*. To define the facility one must know its location (latitude and longitude) and design, including site soils, substructure, structural, and nonstructural components. One creates an inventory of the damageable assemblies and identifies the *EDP*—story drift ratio, member force, etc.—that would cause damage to each assembly.
2. *Hazard analysis*. The objective of this stage is to determine the S_{EBE}, that is, the intensity measure associated with 10% exceedance probability in 5 years. This might be parameterized via any of several intensity measures. For present purposes, let us use the damped elastic spectral acceleration response at the building's small-amplitude fundamental period of vibration, $S_a(T_1)$. It can be calculated via software such as Frankel and Leyendecker (2001), and adjusted to account for site classification such as by using F_a or F_v, as appropriate, from the *International Building Code* (ICC 2000).
3. *Structural analysis*. In this simplification, the structural response to which each damageable assembly is subjected is calculated considering the first-mode spectral response. We denote by ϕ_1 the mode shape of a building at its small-amplitude fundamental period of vibration, T_1. Let the modal excitation and modal mass for the first mode be denoted by L_1 and M_1, respectively. Each damageable assembly is assumed to be sensitive to an *EDP*, characteristic of that assembly type, whose value we denote by *x*, and which can be calculated as a function of ϕ_1, L_1 and M_1. For example, considering one frame direction, the *EDP* for a segment of wallboard partition on the *m*th story would be the interstory drift along that wall line, estimated as

$$x \approx \frac{S_{EBE}}{\omega_1^2}\left(\frac{\phi_{1(m+1)} - \phi_{1m}}{h_m}\right)\frac{L_1}{M_1} \tag{22}$$

where $\omega_1 = 2\pi/T_1, \phi_{1m}$ refers to the component of the fundamental mode shape at floor m, and h_m refers to the height of story m.

4. *Damage and loss analysis*. It is assumed that after an assembly is subjected to a certain *EDP*, it will be in an uncertain damage state D, indexed by $d=0$, 1, 2, ... N_D, where $d=0$ indicates the undamaged state. We assume that the damage states can be sorted in increasing order, either because an assembly in damage state $d=i+1$ must have passed through damage state i already, or because the effort to restore an assembly from damage state $d=i+1$ necessarily restores it from damage state $d=i$. The threshold level of *EDP* causing an assembly to reach or exceed damage state d is uncertain, and is denoted by X_d, whose cumulative distribution function is denoted by $F_{Xd}(x)$. The expected value of the cost to restore a damaged assembly from damage state d is denoted by c_d; it can be calculated by standard construction-cost estimation principles. Then, given the response x to which an assembly is subjected, the mean cost to repair the damageable assembly is

$$\overline{y}(x) = \sum_{d=1}^{N_D} c_d p[D=d|EDP=x] \tag{23}$$

where probability

$$\begin{aligned} p[D=d|EDP=x] &= 1 - F_{X_1}(x) && d=0 \\ &= F_{X_d}(x) - F_{X_{d+1}}(x) && 1 \leq d < N_D \\ &= F_{X_{ND}}(x) && d=N_D \end{aligned}$$

and where $d=0$ refers to the undamaged state. Kustu et al. (1982) presented Equation 23 normalized by the replacement cost of the assembly, and referred to it as a component damage function. We use the nonnormalized form to avoid considering the uncertain replacement cost of the component. (Because construction contractors estimate repair effort directly in terms of labor hours and dollar costs, it is less prone to error to deal with c_d directly, rather than as a fraction of another cost that must also be estimated.) In Beck et al. (2002), we took all capacities as lognormally distributed, using the median and logarithmic standard deviations shown there and in Porter et al. (2002a), so

$$F_X(x) = \Phi\left(\frac{\ln(x/\hat{x})}{\beta}\right) \tag{25}$$

where $\hat{x}$ and β vary by assembly type and damage state, and where $\Phi(\ )$ denotes the cumulative standard normal distribution evaluated at the term in parentheses. Introducing subscript k to index particular assemblies, the expected total cost to repair the facility with N damageable assemblies is given by

$$PFL = (1 + \overline{C}_{OP}) \sum_{k=1}^{N} \overline{y}_k(x_k) \tag{26}$$

where $\overline{C}_{OP}$ refers to contractor's mean overhead-and-profit factor (typically 15% to 20%). Then *EAL* is calculated per Equation 20, as in Method 2.

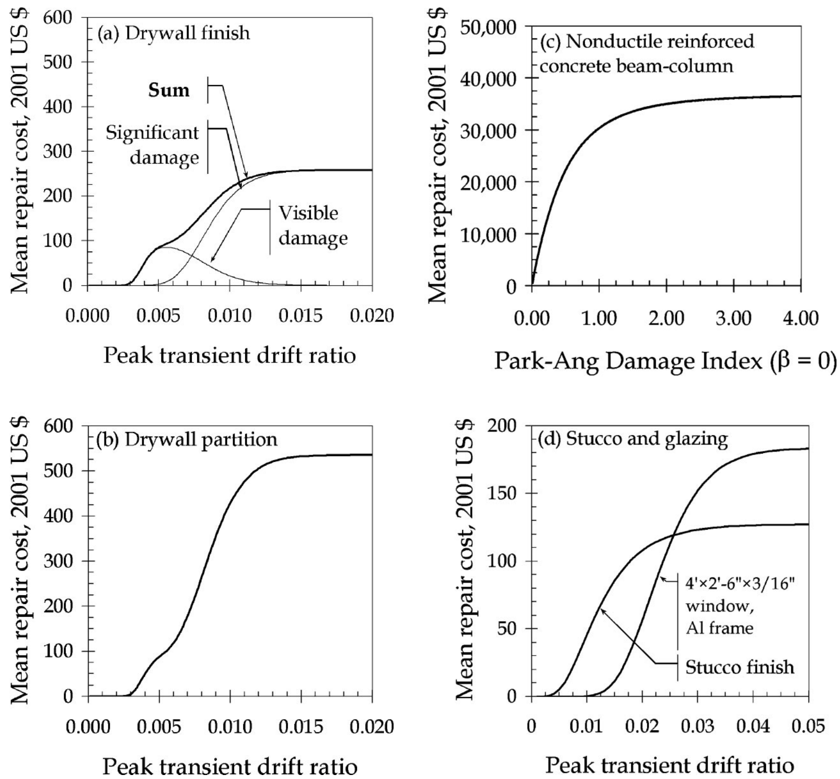

Figure 1. Mean assembly vulnerability functions for five assembly types (d shows two to save space).

For any assembly type, one can create a mean assembly vulnerability function by evaluating Equation 23 as a function of x. These mean assembly vulnerability functions can be archived and reused in later analyses. Figure 1 shows mean assembly vulnerability functions for five damageable assemblies treated in Beck et al. (2002); see that reference for details. Figure 1a shows how Equation 23 produces the overall mean assembly vulnerability function for a 64-sf section of drywall finish, considering two damage states. Figure 1b shows similar results for drywall partition (partition includes the metal studs and drywall finish on one side); Figure 1c does so for nonductile reinforced concrete beam-columns, and Figure 1d shows results for a type of window and a 64-segment of stucco exterior finish.

CASE STUDIES

VAN NUYS HOTEL BUILDING

To compare Methods 1, 2, and 3, we begin with the example of an actual high-rise hotel building located in Van Nuys, California. It is a seven-story, eight-by-three-bay, nonductile reinforced-concrete moment-frame building built in 1966. It suffered damage

Table 1. Summary of assembly fragility parameters and cost distributions (Beck et al. 2002)

				Capacity		Cost, $	
Assembly description	Unit	Limit state; repair	EDP(1)	$\hat{x}$	β	$\hat{x}$	β
Stucco finish, 7/8″, 3–5/8″ metal stud, 16″ OC	64 sf	1. Cracking; patch	PTD	0.012	0.5	125	0.2
Drywall fin., 5/8-in., 1 side, metal stud, screws	64 sf	1. Visible dmg; patch	PTD	0.0039	0.17	88	0.2
Drywall fin., 5/8-in., 1 side, metal stud, screws	64 sf	2. Signif. dmg; replace	PTD	0.0085	0.23	253	0.2
Drywall ptn, 5/8-in., 1 side, metal stud, screws	64 sf	1. Visible dmg; patch	PTD	0.0039	0.17	88	0.2
Drywall ptn, 5/8-in., 1 side, metal stud, screws	64 sf	2. Signif. dmg; replace	PTD	0.0085	0.23	525	0.2
Nonductile CIP RC beam or column	ea	1. Light; epoxy	PADI	0.080	1.36	8000	0.42
Nonductile CIP RC beam or column	ea	2. Moderate; jacket	PADI	0.31	0.89	20500	0.4
Nonductile CIP RC beam or column	ea	3, 4. Severe or collapse; replace	PADI	0.71	0.8	34300	0.37
Window, Al frame, sliding, hvy sheet glass...	ea	1. Cracking; replace	PTD	0.023	0.28	180	0.2
Paint on exterior stucco or concrete	sf	Paint	(2)	N/A		1.45	0.2
Paint on interior concrete, drywall, or plaster	sf	Paint	(2)	N/A		1.52	0.2

(1) PTD=peak transient drift ratio; PADI=$(\phi_m-\phi_y)/(\phi_u-\phi_y)$, where ϕ_m=maximum curvature, ϕ_y=yield curvature, ϕ_u=curvature at maximum moment

(2) Paint entire room, hallway, etc. to achieve reasonable uniform appearance if any component requires painting.

in the 1971 San Fernando earthquake and more extensive damage in the 1994 Northridge earthquake, after which it was seismically upgraded. We analyzed the building in its pre-Northridge condition. See Beck et al. (2002) and Porter et al. (2002a) for details of the hazard model, structural model, component capacity distributions, and unit repair costs. We examined 20 levels of ground motion: $S_a(1.5\text{ sec},5\%)=0.1g,\ 0.2g,\ \ldots\ 2.0g$. At each S_a level, we selected 20 ground-motion time histories at random (within scaling limitations and other preferences) from 100 provided by Somerville et al. (1997), randomly pairing each with a sample of the stochastic structural model to perform a nonlinear time-history structural analysis. In each of the 400 structural analyses, all structural, damage, and cost parameters varied according to prescribed probability distributions.

Masses were taken as perfectly correlated, normally distributed, with coefficient of variation equal to 0.10, as suggested by Ellingwood et al. (1980). Damping was taken as normally distributed with mean value of 5% and coefficient of variation equal to 0.40, as derived in Beck et al. (2002). Structural members were taken as having deterministic

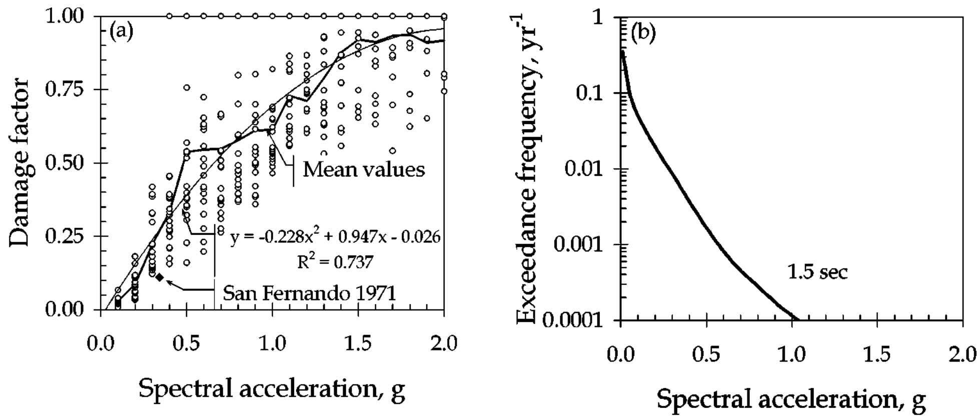

Figure 2. (a) Mean seismic vulnerability function, and (b) site hazard function for Van Nuys building.

stiffnesses (including post-yield, unloading, etc.) but with yield and ultimate force and deformations that are perfectly correlated, normally distributed, with coefficient of variation of 0.08, as suggested by Ellingwood et al. (1980).

Component capacities were taken as lognormally distributed, with median (denoted by $\hat{x}$) and logarithmic standard deviation (denoted by β) summarized in Table 1. By "component capacity," we mean the uncertain value of the engineering demand parameter (*EDP*) at which a component exceeds a limit state. Limit states are defined in terms of the required repairs. Repair-cost distributions for individual damaged components (referred to here as unit-repair costs) were taken as lognormally distributed with median ($\hat{x}$) and logarithmic standard deviations (β) summarized in Table 1, with mean values estimated by a professional cost estimator. Contractor overhead and profit were taken as uniformly distributed between 15% and 20% of total direct costs (the sum of the costs to repair individual assemblies). Unit costs are in 2001 U.S. dollars.

Two limitations of the model should be acknowledged. First, it does not fully capture collapse. Second, it employed uncoupled structural and damage analyses, that is, damage was taken as conditionally independent of structural characteristics, conditioned on structural response. Recent research suggests that such an uncoupled analysis can underestimate uncertainty in repair costs, among other effects (Shaikhutdinov et al. 2004).

The resulting seismic vulnerability function is shown in Figure 2a. The *x*-axis represents 5% damped elastic spectral acceleration (denoted by S_a) at the building's small-amplitude fundamental period, 1.5 sec. The *y*-axis measures repair cost as a fraction of replacement cost. Each circle represents one loss simulation. The jagged line indicates mean damage factor at each S_a level. The smooth curve is a polynomial fit to all of the data. Each simulation includes one nonlinear time-history structural analysis using one ground-motion time history, one simulation of the (uncertain) mass, damping, and force-deformation characteristics of the building, one simulation of the damageability of each of 1,233 structural and nonstructural components, and one simulation of the unit-repair

Table 2. Approximation of earthquake loss using probable frequent loss (*PFL*)

	Van Nuys
S_{NZ}	0.05g
S_{EBE}	0.20g
$G(S_{NZ})$, yr^{-1}	0.1026
$G(S_{EBE})$, yr^{-1}	0.0195
H, yr^{-1}	0.0617
PFL Methods 1 and 2	$613,000
Method 3	$930,000
EAL Method 1	$53,600
Method 2	$37,800
Method 3	$57,400

cost for each of nine combinations of component type and damage state. The analysis included 20 simulations for each of 20 S_a increments from 0.1 g to 2.0 g. The 400 nonlinear time-history structural analyses took approximately 12 hours of computer time on an ordinary desktop computer; the subsequent loss analysis took less than an hour. The most time-consuming portion of the analysis was creating the structural model.

The jaggedness of the mean-vulnerability curve in Figure 2a reflects three effects. First, beam and column repair costs begin to saturate near S_a=0.5 g for some simulations, possibly because of plastic hinges acting as structural fuses. Second, the damage factor begins to saturate near S_a=0.4 g. Repair cost was capped at the replacement cost of the building, and costs were estimated to reach or exceed this value in some simulations beginning at S_a=0.4 g. Third, with a residual coefficient of variation of damage factor as high as 0.50, one would expect to see some jaggedness in the mean vulnerability function from a Monte Carlo simulation with 20 samples per S_a level.

Figure 2b provides the site seismic hazard function, denoted by $G(S_a)$ and defined as the mean annual exceedance rate of ground shaking as a function of S_a. We used Frankel and Leyendecker (2001) to calculate the hazard at T=1.0 and 2.0 sec, with soil at the B-C boundary, and then interpolated in the log-frequency domain to calculate the hazard at T=1.5 sec, using *International Building Code* adjustments to account for soil condition. Note that Figure 2a shows that for S_a up to about 0.5 g, a linear approximation for the mean damage factor $y(s)$ is reasonable; beyond 0.5 g, Figure 2b shows that $v(s)$ is so small that the integrand of Equation 1 makes little contribution. This observation motivated the linear approximation for $y(s)$ introduced in Equation 11.

Using the detailed seismic vulnerability function and mean site shaking hazard function of Figure 2, we calculated *EAL* by Method 1. Equation 9 produces a value of *EAL*=$54,000 for $S_a \leq 2.0$ g, where the remainder term, R, has an (negligible) upper bound of $37. For purposes of evaluating *EAL* under Methods 2 and 3, we take S_{NZ} =0.05 g and $S_{EBE} \approx 0.2$ g. Table 2 compares the values of *PFL* and *EAL* calculated using the three methods. (Note that *PFL* for Method 2 is taken from the ABV analysis of Method 1. The difference between the *PFL* values for Methods 1 and 3 is due to the

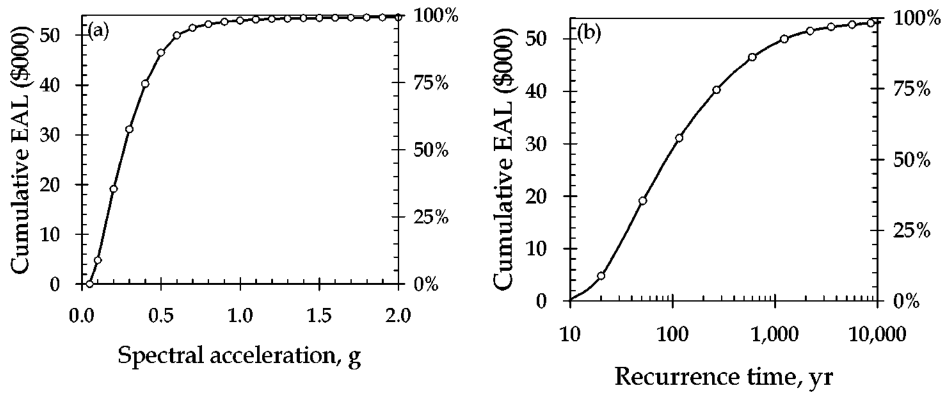

Figure 3. Dominance of frequent events in expected annualized loss for Van Nuys building.

linear approximation of structural response.) Agreement is reasonable: Methods 2 and 3 produce *EAL* estimates within about 30% of that of Method 1. That Method 3 produces a reasonable estimate is particularly promising: at least in this case, one need not create a nonlinear structural model to get a reasonable estimate of *PFL* and *EAL*.

We performed three additional tests of EBE and Method 2. First, we evaluated Equation 9 at each of $n=1, 2, \ldots 20$, for $\Delta s=0.1$ g. The resulting plot Figure 3 shows the cumulative contribution to *EAL* considering only $S_a \leq 0.1$ g, then $S_a \leq 0.2$ g, etc. Figure 3a shows the results plotted against S_a, while Figure 3b shows the same information plotted against mean recurrence time. Observe that only about 15% of cumulative economic loss comes from events such as the *PML*-level shaking or greater ($S_a > 0.5$ g). As important as the 500-year earthquake is as a design basis for life safety, it is largely irrelevant here for economic considerations. Almost half the expected losses for this building result from shaking of $S_a \leq 0.25$ g, i.e., events with a recurrence time of 85 years or less. Approximately 35% of loss is due to $S_a \leq S_{EBE}$. Ideally, cumulative loss from $S_a \leq S_{EBE}$ would always be near 50%, which would suggest that S_{EBE} is a good representative scenario shaking level, but of course the fraction will likely vary between buildings, so a cumulative *EAL* fraction of 35% at the S_{EBE} defined this way seems acceptable.

CUREE-CALTECH WOODFRAME PROJECT BUILDINGS

As a second test of EBE and of Method 2, we compared Methods 1 and 2 using 14 hypothetical (but completely designed) buildings from the CUREE-Caltech Woodframe Project (Porter et al. 2002b). The buildings are variants of four basic designs referred to as index buildings (Reitherman and Cobeen 2003). The index buildings include a small house (single story, 1,200 sq. ft., stucco walls, no structural sheathing), a large house (two stories, 2,400 sq. ft., some walls sheathed with plywood or OSB, stucco exterior finish), a three-unit townhouse (two stories, 6,000 sq. ft. total, some walls sheathed with plywood or OSB, stucco exterior finish), and an apartment building (three stories, 13,700 sq. ft., 10 dwelling units, and tuck-under parking). Each index building included four or more variants: a poor-quality version, a typical-quality version, a superior-quality

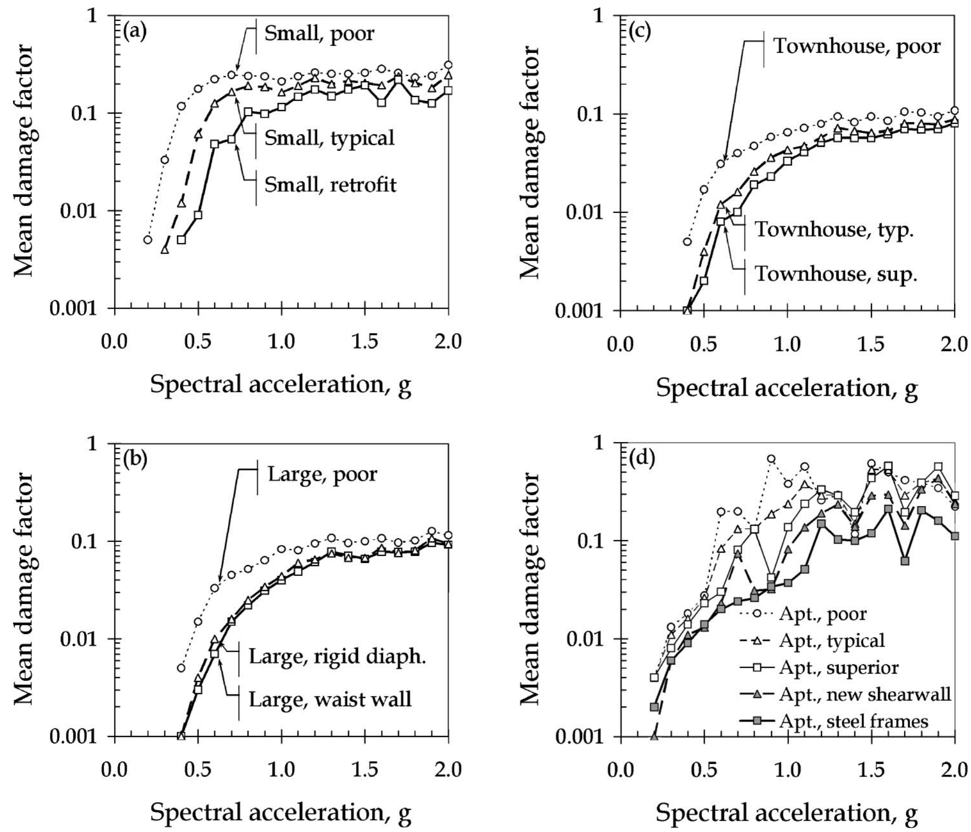

Figure 4. CUREE-Caltech Woodframe Project mean vulnerability functions.

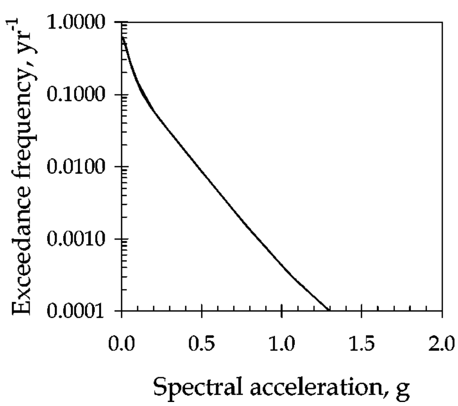

Figure 5. Seismic hazard function for a Los Angeles site.

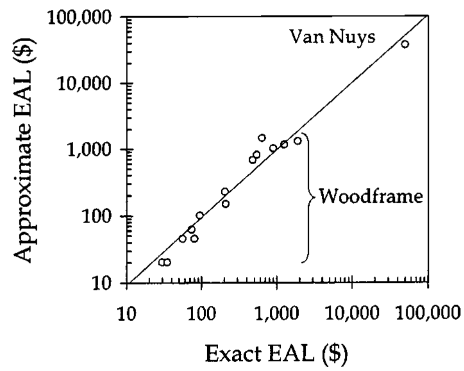

Figure 6. Comparing *EAL* by Methods 1 and 2 for 15 sample buildings.

version, and one or more alternative designs or retrofits. We considered these woodframe buildings located at an arbitrary site, chosen to be in Los Angeles, California, at 33.9°N, 118.2°W. Using Frankel and Leyendecker (2001) to determine site hazard, adjusting for NEHRP soil category D, we find $S_{EBE}=0.4$ g. Of the 19 buildings examined in Porter et al. (2002b), 14 have nonzero loss estimates at S_{EBE}. Their seismic vulnerability functions are shown in Figure 4. They are shown with a logarithmic *y*-axis because of the low losses suffered at lower shaking intensities. The site hazard is shown in Figure 5. The jaggedness of some of the vulnerability functions reflects sensitivity to collapse.

Figure 6 shows the *EAL* values for these 14 woodframe buildings and for the Van Nuys building calculated by Method 1 (referred to in the figure as "exact") and by Method 2 (referred to as "approximate"), using EBE as defined above. We denote *EAL* estimated under Method 1 by EAL_1, define estimation error as

$$\varepsilon \equiv \frac{EAL_2 - EAL_1}{EAL_1} \tag{27}$$

and take the error for each case-study building as a sample of ε. We find the sample mean and sample standard deviation of this error are $\overline{\varepsilon}=0.12$ and $s_{\varepsilon}=0.52$, respectively. Thus, for this sample of 15 buildings, the use of S_{EBE} defined as the shaking with 10% exceedance probability in 5 years produces a fairly modest (12%) error in the estimate of *EAL*, relative to the "exact" method that requires analysis of the complete seismic vulnerability function.

As a final test, we calculated the error if one defines S_{EBE} as shaking with 50% exceedance probability in 50 years, and found $\overline{\varepsilon}=0.06$ and $s_{\varepsilon}=0.47$. Defining EBE this way produces slightly more accurate results for the case-study buildings than using shaking intensity with 10% exceedance probability in 5 years (as we have done), although at the cost of meaningful risk communication.

The *EAL* values shown in Figure 6 might be quite meaningful to the real-estate investor. In the case of the pre-Northridge Van Nuys building, whose replacement cost is approximately $7.0M and whose annual net operating income is on the order of $1M, an

EAL of \$54,000 represents a significant expense. The *EAL*s for the poorer-performing woodframe buildings can exceed \$1,000 annually. This would be a significant expense for a small investor, of the same order as homeowner insurance (Insurance Information Institute 2003). Investors might be interested in knowing and including these figures in their financial analysis during the due-diligence phase.

USING SEISMIC RISK IN REAL-ESTATE INVESTMENT DECISION MAKING

The previous sections have suggested that *PFL* and *EAL* can be calculated for an individual facility using linear assembly-based vulnerability, together with a site economic hazard coefficient *H* that can be mapped or tabulated. It was suggested that an investor could use *PFL* as a meaningful scenario loss and *EAL* as an operating expense to account for seismic risk during a standard financial analysis of an investment opportunity. It was shown that *EAL* can be estimated based on *PFL* and on *H*, that *EAL* in the cases discussed here represents a significant expense worthy of consideration in a financial analysis, and that, if considered, *EAL* would help to make seismic risk a market force.

When one examines seismic risk in the context of the larger investment decision, some additional questions arise: how important is the uncertainty of future earthquake repair costs, compared with the market-related uncertainties of investment value? How can one treat both sources of uncertainty in a rigorous decision-making methodology?

We have mentioned some of the sources of uncertainty of investment value, most notably future market rents and vacancy rates. We refer to this uncertainty as market risk, and it can be large. Holland et al. (2000) inferred the volatility of real-estate return from volatility of commercial mortgage interest rates along with other observable variables, and from the standard deviation of daily rates of return on equity real-estate investment trusts (REITs), as part of a larger study of how uncertainty affects the rate of investment. They found that the implied volatility of the capitalization rate for commercial real estate (i.e., the standard deviation of the difference between return in two successive years) is on the order of 0.15 to 0.30. Depending on how one models the long-term effects of volatility, and how much information one assumes the investor has, the coefficient of variation (denoted here by *COV*) of property value can exceed 6, a very high value (Beck et al. 2002)! Even if the "true" uncertainty of market value were an order of magnitude less, say, having a *COV* of 0.5 to 1.0, this would mean that market uncertainty could swamp uncertainty of future earthquake economic losses. Later, we test this conjecture using the Van Nuys building as a case study.

How can information about uncertainty in property value be used to inform an investment or risk-management decision? A well-established discipline of economics called decision analysis deals explicitly with high-stakes decisions under conditions of substantial uncertainty (e.g., Howard and Matheson 1989). Briefly, in a decision-analysis approach, a decision is framed as a situation in which a decision maker chooses between two or more mutually exclusive alternatives, each of which can have uncertain outcomes measured in economic or other value terms. The decision maker's preferences are encoded in a utility function, a relationship between value outcome (e.g., the decision maker's uncertain future wealth, denoted here by x) and an abstract parameter called *utility* (denoted here by u) that can be thought of as quantifying the desirability of possible outcomes, with more utility meaning a more desirable outcome. The preferable alternative is the one that offers the highest expected value of utility, $E[u]$.

Because the utility function is monotonically increasing in x, the preferable alternative is also the one that offers the highest value of certainty equivalent (CE), defined as the inverse of the utility function evaluated at the expected value of utility, so $CE = u^{-1}(E[u])$. If the utility function measures the desirability of a monetary outcome, then CE has units of money, and is equivalent to the amount of money one should accept for certain in exchange for an uncertain bet. In other words, CE measures in money terms what the bet is worth to the decision maker, considering the uncertainty of the outcome and his or her risk attitude. A convenient idealization of a decision maker's utility function is the exponential form

$$u(x)=1-\exp(-x/r) \tag{28}$$

where $u(x)$ is the decision maker's utility of the wealth state x and r, referred to as risk tolerance, is a constant in units of money that reflects the decision maker's risk attitude. A larger value of r means the decision maker more closely approaches a risk-neutral attitude, where only the expected value of x is considered. This form of the utility function has the interesting feature that, if x represents change in the decision maker's wealth state rather than absolute wealth state, decisions that optimize $E[u]$ or CE still satisfy the rules of decision analysis, meaning that one can examine an uncertain deal in isolation, ignoring other effects on the decision maker's wealth.

We can now discuss how decision analysis can address market risk and uncertainty in earthquake loss to select an optimum decision alternative that maximizes CE. In the case of a real-estate investment opportunity, the CE of purchasing a property is a function of uncertain future net income stream (rental and other income less operating and other expenses besides earthquake repair costs), the purchase price, uncertain future earthquake repair costs, the variance of net income and of earthquake repair costs, and the decision maker's risk tolerance, r. We denote by I the uncertain present value of the net income stream; it is typically quantified during the investor's financial analysis of a potential purchase. We denote by L the uncertain present value of future earthquake repair costs. One can show that CE can be expressed as (Beck et al. 2002):

$$CE=E[I]-C_0-E[L]-\frac{Var[I]+Var[L]}{2r}-rR \tag{29}$$

where

$E[I]$	=	expected present value of the future net income stream
C_0	=	purchase price
$E[L]$	=	expected present value of future seismic losses
$Var[I]$	=	variance of the present value of the net income stream (a measure of market risk)
$Var[L]$	=	variance of the present value of future seismic losses

r = risk tolerance of Equation 28

R = remainder terms associated with higher-order moments of income and seismic loss

If market risk indeed produces a coefficient of variation of at least 0.5 to 1.0, as implied by our analysis of the Holland et al. (2000) study, then the variance term is dominated by market risk, not only for the Van Nuys building but probably for most commercial investment properties. The remainder rR is small compared with $E[L]$ for most decision makers. Recognizing that $E[L]$ for a planning period of t and a risk-free real interest rate of i is simply

$$E[L]=EAL\cdot\left(\frac{1-\exp(-it)}{i}\right)=H\cdot PFL\cdot\left(\frac{1-\exp(-it)}{i}\right) \tag{30}$$

then the certainty equivalent of Equation 29 can be approximated by

$$CE=E[I]-C_0-H\cdot PFL\cdot\left(\frac{1-\exp(-it)}{i}\right)-\frac{Var[I]}{2r} \tag{31}$$

The first three terms in Equation 31 are the risk-neutral part, i.e., the value of CE if the risk tolerance $r\rightarrow\infty$. This portion can be calculated in the standard financial analysis and due-diligence study that the bidder undertakes. Variance of income can be estimated based on studies such as Holland et al. (2000) or using the investor's judgment. Notice from Equation 31 that increasing market uncertainty or decreasing risk tolerance reduce the certainty equivalent of a real-estate investment below its risk-neutral value, as might be expected.

In Beck et al. (2002), we present a methodology for eliciting decision-maker risk tolerance. In a study of six U.S. and four Japanese investors, we found that one can estimate r as a function of the investor's annual budget or the size of investments he or she typically makes. An investor in the Van Nuys building would have r on the order of \$100M.

We use our knowledge of r, EAL, and market risk to calculate CE for the Van Nuys building under four investment alternatives: (1) do not buy; (2) buy and do not mitigate seismic risk ("as-is"); (3) buy and purchase earthquake insurance; or (4) buy and perform a seismic retrofit. The retrofit involved adding shearwalls to the structural system; see Beck et al. (2002) for the modified structural model, hazard, and resulting vulnerability function. Assuming a risk-free real discount rate of 2%, an annual insurance premium equal to 3.5% of the insured limit, and a capitalization rate of 0.13 (pre-tax annual net income as a fraction of purchase price), we found that the CE for alternative 2 was the greatest of the four alternatives, meaning that the best choice of the four is to buy and not to retrofit (Table 3). Note how small uncertainty in earthquake loss is compared with uncertainty in income.

We also performed sensitivity studies, varying the discount rate, risk tolerance, variance of income, and the price of insurance over reasonable bounds, and found that under most conditions, the best choice was to buy and not to retrofit. For conditions of low risk tolerance ($r<25$ million) or high uncertainty of net income ($COV[I]>2$), the best choice

Table 3. Certainty equivalent of four investment alternatives. All figures in $M.

	Don't buy	As-is	Insure	Retrofit
Mean after-tax PV of income, $E[I]$	$0.0	$39.0	$31.5	$39.0
Purchase price C_0	0.0	10.0	10.0	12.4
EAL	0.0	0.054	0.033	0.043
After-tax PV of earthquake loss $E[L]$	0.0	1.6	1.0	1.3
Variance of income Var$[I]$	0.0	1521.0	1521.0	1521.0
Variance of earthquake loss, Var$[L]$	0.0	0.9	0.7	0.7
Certainty equivalent *CE*	0.0	19.8	12.9	17.7

was not to buy. We did not identify any conditions under which it was preferable to buy insurance or to seismically retrofit the building. Note, however, that we did not treat the risk of injury or death from earthquakes, which could make a material difference in an investment decision.

The reader might feel that earthquake loss prevention is or should be more than just economics, and should be treated as a community issue for which additional incentives should be offered. Perhaps so, but such decisions should be based on careful consideration of the retrofit cost, of the cost of the incentives, of the value of casualties avoided, and of competing uses for these funds for life safety.

CONCLUSIONS

Through a case study of a nonductile reinforced-concrete moment-frame building, we have shown that probabilistic repair costs can be dominated by small, frequent events, as opposed to rare, *PML*-level losses. Using this concrete building and 14 additional woodframe buildings, we have also shown that expected annualized loss (*EAL*) is approximately proportional to a scenario loss referred to as the probable frequent loss (*PFL*), defined similarly to *PML*. The constant of proportionality, referred to here as the site economic hazard coefficient (*H*), can be mapped or tabulated for ready use by structural engineers or investors. *PFL* is defined as the expected value of loss conditioned on the occurrence of shaking with 10% exceedance probability in 5 years. This is the economic-basis earthquake, EBE, named and defined intentionally similar to the design-basis earthquake (DBE) of older codes. We have shown that a simplified loss-analysis approach, referred to as linear assembly-based vulnerability (LABV), can produce a reasonable estimate of *PFL* and consequently *EAL*. We have shown that, from a real-estate investor's viewpoint, uncertainty in earthquake repair cost can be negligible compared with uncertainties arising from real-estate market volatility.

There are several interesting implications of these findings for seismic risk management. One of the most common opportunities for seismic risk management is the bidding phase just prior to the purchase of commercial real estate in seismically active regions. Common practice currently produces little information that actually helps investors consider seismic risk in their investment decision. Consequently, this most-common opportunity for risk management is usually a missed opportunity. This problem might be alleviated using *PFL* rather than (or in addition to) *PML*. *PFL* offers several advantages as a metric of investment performance:

1. It better reflects investors' typical planning period and would be more meaningful as an upper-bound loss than *PML*, which tends to reflect too rare an event to the investor.
2. *EAL* is proportional to *PFL* through a site economic hazard coefficient *H*, which can be mapped.
3. *EAL* can be used as an operating expense in the investment financial analysis to reflect seismic risk and thus to make seismic risk more of a market parameter.
4. Because of its similarity to *PML*, *PFL* should be readily understood by engineers and investors, and could be calculated during the due-diligence phase of the bidding process.
5. At shaking levels addressed by the *PFL*, it is likely that linear structural analysis can be used to estimate loss with acceptable accuracy. User-friendly software exists to perform linear structural analysis quickly; with practical extensions, this type of software could be used to calculate *PFL* and *EAL* inexpensively, within the budget of a due-diligence study, and therefore competitively with loss-estimation software that relies on expert opinion.
6. In the case studies, defining S_{EBE} as the level of shaking with 50% exceedance probability in 50 years was shown to improve the accuracy of the *EAL* approximation slightly, but at the cost of meaningful risk communication for the investor.

Finally, this paper has shown how formal decision analysis can be used in the investment decision process to account for real-estate market risk, future earthquake losses, and the investor's risk attitude, and to choose among competing risk-management alternatives based on the maximum certainty equivalent. The decision-analysis approach requires the additional information of variance of market value and the decision maker's risk tolerance. Variance of earthquake repair costs is not needed, since it makes a negligible contribution in the decision analysis compared with the uncertainty in market conditions.

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ABBREVIATIONS

ABV	assembly-based vulnerability
c_d	mean cost to repair one unit of an assembly from damage state d
C_0	initial cost
CE	certainty equivalent
C_{OP}	factor applied to total direct construction cost to account for contractor overhead and profit
d	particular value of damage state
D	uncertain damage state
DBE	design-basis earthquake
EAL	expected annualized loss
EBE	economic-basis earthquake
EDP	engineering demand parameter
FEMA	Federal Emergency Management Agency
$F_X(x)$	cumulative distribution function of uncertain variable X evaluated at x
$G(s)$	mean annual frequency of exceeding s
G_{EBE}	mean annual exceedance frequency of S_{EBE}.
G_{NZ}	mean annual exceedance frequency of S_{NZ}
G_U	mean annual exceedance frequency of S_U
H	economic hazard coefficient
h	story height
i	discount rate
I	income
L	loss
LABV	linear assembly-based vulnerability
m	slope of $\ln(G(s))$
N_D	number of possible damage states
NIBS	National Institute of Building Sciences
PFL	probable frequent loss
PML	probable maximum loss
r	risk tolerance
s	seismic intensity
S_{EBE}	seismic intensity associated with the economic-basis earthquake

S_{NZ}	seismic intensity associated with initiation of loss
S_U	seismic intensity associated with saturation of loss
T_1	small-amplitude fundamental period of vibration
$u(x)$	utility function evaluated at x
V	value exposed to loss
$v(s)$	absolute value of the first derivative of $G(s)$
$\overline{y}(s)$	mean seismic vulnerability function evaluated at s
y_U	upper-bound loss
Γ	modal participation factor
ϕ_1	fundamental mode-shape vector
ω_1	fundamental frequency of vibration

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